METHOD OF SPEEDING UP BUILDINGS TIME HISTORY ANALYSIS BY USING APPROPRIATE DOWNSAMPLED INTEGRATION TIME STEP

Peng Zhong¹ and Farzin Zareian²

ABSTRACT

This paper summarizes the research findings on identifying an appropriate time step (i.e., sampling frequency) for ground motion acceleration time-history in order to maximize computational efficiency and ensure accurate estimation of structural system’s displacement response. With increases in number of degrees of freedom of structural systems, an issue commonly encountered in tall structures, the nonlinear analysis using ground motions with small sampling time becomes quite time consuming. This study proposes a viable method to reduce the analysis time by downsampling ground motion data. Downsampling is achieved by increasing the ground motion’s time step which in turn reduces the overall number of data points. In this method a structure’s Frequency Response Function is used to judge the appropriate time step according to the characteristics of a given ground motion and structural system. Filtering and downsampling techniques are applied to the original ground motion to generate a downsampled ground motion with the goal of obtaining a highly efficient time history analysis without significant error. This paper shows a practical method and delivers the associated tools for proper downsampling of ground motion’s acceleration time-history to be used in response history analysis. The method is not only practical, but also capable of controlling errors in estimates of response. Results of the proposed method are judged in terms of a goodness of fit test, comparing observed responses to the downsamled ground motions with similar estimates using original ground motions for typical SDOF and MDOF structural systems.

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This paper summarizes the research findings on identifying an appropriate time step (i.e., sampling frequency) for ground motion acceleration time-history in order to maximize computational efficiency and ensure accurate estimation of structural system’s displacement response. With increases in number of degrees of freedom of structural systems, an issue commonly encountered in tall structures, the nonlinear analysis using ground motions with small sampling time becomes quite time consuming. This study proposes a viable method to reduce the analysis time by downsampling ground motion data. Downsampling is achieved by increasing the ground motion’s time step which in turn reduces the overall number of data points. In this method a structure’s Frequency Response Function is used to judge the appropriate time step according to the characteristics of a given ground motion and structural system. Filtering and downsampling techniques are applied to the original ground motion to generate a downsampled ground motion with the goal of obtaining a highly efficient time history analysis without significant error. This paper shows a practical method and delivers the associated tools for proper downsampling of ground motion’s acceleration time-history to be used in response history analysis. The method is not only practical, but also capable of controlling errors in estimates of response. Results of the proposed method are judged in terms of a goodness of fit test, comparing observed responses to the downsampled ground motions with similar estimates using original ground motions for typical SDOF and MDOF structural systems.

Introduction

Nonlinear response history analysis has proven to be a useful tool in earthquake engineering research and seismic design practice. With the exponentially rapid development of computers, its use in engineering practice has become even more prominent. However, for analytical models with many degrees of freedom, especially tall structures, nonlinear response history analysis using ground motions with a small sampling time step (high sampling rate) becomes quite time consuming. A ground motion time history is a representation of an earthquake. Downsampling the time history by reducing the sampling time step (the number of data points) is an elegant way to achieve computational efficiency. Simply put, computation time is saved by reducing the number of steps in the analysis. Clearly some errors in the estimates of structural response will be induced by changing a ground motion record in this way. Hence, it is natural to investigate if the

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gain in computational efficiency can outweigh the loose of accuracy. Motivated by this goal, this research seeks to identify an appropriate and efficient time step, given a ground motion and structural system, which can guarantee there is enough data to execute an accurate analysis with an acceptably small and stable error. With careful consideration of efficiency and accuracy, this research proposes a ground motion downsampling method that would be applicable to structural analysis of a wide range of structural systems and ground motions.

Time-frequency analysis and filtering have been well-studied for over a hundred years. However, the application of downsampling to ground motion signals has seldom been studied. (Todorovska, et al, 2009) studied reduced representation of strong ground motion records as a superposition of a relatively small number of pulses. Such representation is obtained by the expansion of velocity in orthogonal wavelet series using the Fast Wavelet Transform, and approximation by only the largest energy terms in the series. With respect to the goals of the research proposed in this paper, there are three deficiencies in aforementioned reference: (i) ground motions are just downsampled in different percentages rather than establishing a criteria to determine an optimum time step with consideration to efficiency and accuracy; (ii) just 4 cases of nonlinear Single Degree of Freedom (SDOF) systems are utilized in the study, making it hard to determine the method’s applicability to other structures, (i.e. structures with different periods and ductility, and MDOF structures with higher mode contributions such as tall buildings); and (iii) the main focus of the study is comparing the difference of ground motions energy and collapse time of structure systems, whereas, engineers and scientists are also interested in predicting seismic response parameters commonly generalized as Engineering Demand Parameters (EDPs), such as maximum displacement, interstory drift, and peak floor acceleration.

The objective of this work is to obtain a practical method to synthesize ground motions using a downsampling. In this framework, this paper examines how far one can downsample a ground motion and still maintain the accuracy of similar linear and nonlinear displacement response. In addition, we propose how to estimate an optimum time step, considering variation of structural systems and ground motions. The exact manner that frequencies impact the structural response is beyond the scope of this paper; the method is shown to be successful in limiting error for a large data set of ground motions in most cases. Where error exceeded the desired target, it still remains acceptably small for an engineering application of response history analysis.

**Theoretical Background of Ground Motion Signal Processing**

**Downsampling**

In signal processing, downsampling (or ‘subsampling’) is the process of reducing the sampling rate of a signal. This is usually done to reduce the data rate or the size of the data by preserving every n data point and delete the points in between. Here, n is the downsampling factor, which is usually an integer or a rational fraction greater than unity. This factor times the sampling time or, equally, divides the sampling rate. For example, if ground motion at 200Hz is downsampled to 100Hz, the time step is increased from 0.005s to 0.1s. The ground motion was therefore downsampled by a factor of 2. Since downsampling reduces the sampling rate, we must be careful to make sure the Shannon-Nyquist sampling theorem criterion is maintained. If the sampling theorem is not satisfied, then the resulting digital signal will have aliasing. The Nyquist-
Shannon sampling theorem (Oppenheim V.A, & Schafer W.R, 1989) is the fundamental theorem in the field of information theory, which states that “When sampling a signal, changing time step to be a larger one, the sampling frequency must be greater than twice the bandwidth of the input signal in order to be able to reconstruct the original perfectly from the sampled version.”

If the sampling theorem is not satisfied, then frequencies will overlap. This overlap is called aliasing, referring to an effect that causes different signals to become indistinguishable. In this case, the unique characteristic of ground motion is changed, and the amplitude in particular frequency is different from the original ground motion. In essence, aliasing causes a downsampled ground motion to lose meaning, i.e. the acceleration or time history is no longer a good representation of the original data; too much information has been thrown away. Figure 2.1a shows a signal that has an acceptable time step and perfectly obeys the Nyquist-Shannon sampling theorem; and Figure 2.1d shows signal has a large time step, which makes the signal overlap in frequency domain and finally become undistinguishable.

To ensure that the sampling theorem is satisfied, a low-pass filter is used as an anti-aliasing filter to reduce the bandwidth of the signal before the signal is downsampled; the overall process (low-pass filter, then down-sample) is called decimation. The low pass filter should be implemented to eliminate useless higher frequencies and avoid aliasing. After that, one can safely reduce data by picking out data in certain interval.

**Frequency Response Function**

Fundamentally, a frequency response function (FRF) is a mathematical representation of the relationship between the input and output of a system, which could be considered as a transfer function expressed in frequency domain, see Figure 2.2. An FRF expresses the structural response
to an applied force as a function of the forces frequency. The response may be given in terms of
displacement, velocity, or acceleration. This study is limited to the input and output being the base
acceleration and the structural displacement response, respectively.

Figure 2.2. Frequency Response Function Conceptual Representation

FRF is a linear function with no limitation in application for linear structural models. However, once the structures goes through nonlinear action, its period elongates (frequency
reduction), therefore, an analysis time step based on linear behavior should usually be short enough
for calculating inelastic response. We still can use this approach as the basis to accomplish efficient
structural response estimation. For a linear SDOF system, FRF is given by (Ewins D.J., 2000):

\[ H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{(k - \omega^2 m) + i(\omega c)} \]  
(2.3)

Where \( k, m, c \) are stiffness, mass and damping coefficient of SDOF system. \( \omega \) is the
forcing frequency in rad/s, which is varied over the frequency range of interest. Hence:

\[ F(\omega) \ast H(\omega) = X(\omega) \]  
(2.4)

Using Eq. (2.4), the structural system response in frequency domain is easily obtained. The
biggest advantage of using the FRF method is that we can estimate the structural response without
any complicated time domain structural analysis. Through this method, one can predict the
minimum required frequency content with which the response is preserved. Using this minimum
required frequency we can decide on the largest appropriate time step, which is capable of
tremendously reducing the cost of structural analysis (i.e., time) and making the process efficient.

For an MDOF system, we can obtain force and displacement Frequency Response Function
using Eq. (2.5) (Roy, R.C., 2006). The complex FRF in physical coordinates, \( H_{ij}(\omega) \), gives the
response at coordinate \( u_i \) due to unit harmonic excitation at \( P_j \).

\[ H_{ij}(\omega) = \sum_{r=1}^{N} \phi_i \phi_j \frac{1}{K_r (1 - r_r^2) + i(2\xi r_r)} \]  
(2.5)
Where $N$ is the number of stories; $\omega_r$ is $r^{th}$ natural frequency; $\xi_r$ is $r^{th}$ modal damping ratio; $r_r$ is the modal frequency ratio given by $r_r = \omega_r / \omega_r$. Modal masses and modal stiffnesses are shown with $M_r$, and $K_r = \omega_r^2 M_r$, respectively. $\phi_{ij}$ is the element in row $i$ of the $r^{th}$ mode $\phi_r$, that is, the element in row $i$ and column $r$ of the modal matrix $\Phi$. According to (A.K. Chopra, 2007), the ground motion can be replaced by the effective earthquake forces, see Figure 2.3.

![Modal masses and modal stiffnesses](image)

Figure 2.3. Effective earthquake forces in response-history analysis

$$ P_{eff} = -m1\ddot{u}_g $$  \hspace{1cm} (2.6)

From Equations (2.5) and (2.6), FRF is obtained:

$$ H_f(\omega) = \sum_{r=1}^{N} H_j m_r $$  \hspace{1cm} (2.7)

$$ H_f(\omega) = \sum_{r=1}^{N} \phi_{jr} \frac{\sum_{q=1}^{N} (\phi_{qj} m_q)}{K_r (1 - r_r^2) + i(2\xi_r r_r)} $$  \hspace{1cm} (2.8)

Where, $H_f$ is the ground motion acceleration and the $f^{th}$ story response displacement FRF.

**Proposed Method for Reducing Ground Motion Sampling Frequency**

The proposed method of generating downsampled ground motions is presented in the following. At first, original ground motion is transferred to frequency domain. With the application of FRF, cutoff frequency is determined. Given the reciprocal relationship between frequency content and sampling time step, the optimum time step for ground motions is obtained. Based on this time step, original ground motion is filtered and then downsampled to obtain the optimum downsampling ground motion.

**Identifying cutoff frequency in SDOF**

Identifying cutoff frequency is a critical step in the proposed method. The optimum time step needs to be found for each analysis case; it is specific to the structure and the ground motion under consideration. Through the FRF method, one can easily foresee the structure response in frequency domain without any time domain structural analysis. With this representation, high
amplitude frequencies are preserved, and low amplitude frequencies can be can be eliminated safely to speed up response history analysis.

Once the FRF method is applied, the amplitude of peak response in the frequency domain can be identified, however, it is diverse and depends on the structural system and target ground motion. Therefore, a situation dependent parameter for determining the highest frequency applicable to all cases is needed. Through trial and error we have identified that the frequency at which 1% of the peak amplitude in the frequency domain is achieved is a stable parameter (deonted as \( f_{1\%} \)). Using \( f_{1\%} \) as a cut off value does not preserve enough data for the FRF to function correctly; it is necessary to consider a higher frequency for cut-off. Again through trial and error the cut off frequency, \( f_{cut} \), was determined to be \( f_{cut}=3f_{1\%} \). The procedure to find the cut-off frequency using this parameter is shown in the following steps:

1) Once FRF is performed for the system, normalize all the frequency amplitude according to maximum amplitude of response (the maximum amplitude usually occurs at or close to structures natural frequency);

2) The largest frequency with a value greater than or equal to 1% of the maximum amplitude is taking to be \( f_{1\%} \). In other words, frequencies that exceed this value do not have amplitude bigger than 1% of the maximum amplitude (in physical terms, the frequencies which exceed \( f_{1\%} \) do not contain information that is particularly valuable to the analysis. The structure does not significantly respond to them.)

3) The Cutoff frequency is taken to be equal to three times the 1% frequency, \( f_{cut}=3f_{1\%} \). (In terms of the FRF function, a certain amount of data is necessary to get accurate results. Through a large number of trials, the coefficient 3 was found which consistently identified a cut-off frequency that also fulfilled the Frequency Response Function)

It is noted that MDOF systems involve multiple FRFs (There is one transfer function for one story output response). The consideration of all of these functions is not needed for determining the optimum time step. Because the earthquake vibration is propagating from the bottom to the top of a structure, high frequency vibrations are reduced along the height of building. Hence, the first story response has the widest frequency bandwidth and includes all the frequency content that is needed for the other stories. Therefore, the FRF of the first story is the critical one and can be used to determine the optimum time step. The procedure to find the cut-off frequency for an MDOF system is the same as the SDOF procedure, and the only difference is the calculation of FRF.

**Filter and downsampling scheme**

As described above, a filter is needed for downsampling. The procedure for applying the filter and downsampling is as follows:

1) Apply low-pass filter to the signal at cutoff frequency \( f_{cut} \) to ensure that the sampling theorem is satisfied.

2) New time step \( dt \) should be less than or equal to the value of \( 1/(2f_{cut}) \) which is suggested to be the proximate multiple of 0.005s.
3) Reduce the data by picking every $m^{th}$ sample: $h(k) = g(mk)$ ($m$ is equal to the value of new time step divided by original time step). Data rate reduction occurs in this step.

Following this procedure, a downsampled ground motion can be generated for each seismic analysis case. The procedure preserves the characteristic of original ground motion that is salient to the analysis, therefore, the accuracy is guaranteed.

**Application of the Proposed Method in Response Assessment of SDOF and MDOF System**

A properly downsampled ground motion is defined as an analogous reduced representation of the original ground motion record such that it preserves the pertinent ground motion information required for analysis. To assess whether this has been fulfilled, a goodness of fit test is employed. This goodness of fit measures the closeness of: (i) maximum displacement response, and (ii) cross-correlation coefficient of entire displacement responses. For all models considered in this study, the Error is considered as the difference between actual and approximated maximum displacements.

$$Error = \frac{\delta_{ori} - \delta_{app}}{\delta_{ori}}$$

(4.1)

Where, $\delta_{ori}$ is the maximum displacement of actual response; $\delta_{app}$ is the maximum displacement of approximated response.

Because FRF applies to linear systems, using the proposed downsampling method for linear SDOF and MDOF systems is expected to produce good results with small error. For nonlinear SDOF and MDOF systems, however, it is not possible for FRF to precisely limit the error associated with the effect that downsampling will have on nonlinear events in the analysis. Period elongation due to nonlinearity will make the natural frequency become smaller. With consideration of FRF, the linear cutoff frequency should be good enough for nonlinear analysis as well. However the impulse delivered to the oscillator during a nonlinear event in the time history depends on the time-step (e.g. changing the length of the time step changes the duration of the impulse and the energy imparted in the deformation of the oscillator). It is assumed that this effect is small but this assumption needs to be verified with time history analysis in order to demonstrate that the proposed method is applicable to nonlinear systems.

**Linear SDOF Results Using Proposed Downsampling Method**

In SDOF models, we consider a range of periods $T = 0.1, 0.5, 1, 2, 3, 4$, seconds with linear viscous damping ratio $\xi = 5\%$. Figure 4.1 shows the statistical representation of errors once the optimum downsampling method is utilized. The black line with a triangle represents the median. The red line with squares represents the 84th percentile (median + 1 standard deviation) and the blue line with stars represents the 16th percentile (median - 1 standard deviation). It can be seen that most of the error (84%) is less than 0.07 and half of the error is smaller than 0.03. For higher periods, the error becomes smaller.
Coefficients of cross-correlation measurement is introduced in this research. It is a quantity that gives the quality of similarity between approximation data and original data. The advantage is that we can compare not only certain data point, but also the whole duration of displacement response. In the proposed methodology, the correlation for displacement response in each case is very high (>0.91) regardless of the SDOF fundamental period. Cross-correlation coefficient between 0.8 and 1 means these two signals are highly similar. Figure 4.2 also illustrates the worst case (coefficient around 0.91) of cross-correlation; we can see that it is still a good approximation.

Figure 4.2. Structure response of lowest cross correlation

Figure 4.3 shows the optimum time steps for downsampling ground motions used from the NGA database in different Periods. The black line with a triangle represents the median. The red line with squares represents the 84th percentile and the blue line with stars represents the 16th percentile (median ± 1 standard deviation). It can be seen that the optimum time step is around 0.05s, whereas most of original time steps are 0.005s. It means the proposed method is 10 times more efficient than the regular one.

Figure 4.3. Optimum ground motion sampling period using the proposed method for reducing ground motion sampling frequency.
Linear MDOF Results Using Proposed Downsampling Method

We test the applicability of the proposed approach for downsampling ground motion records for the analysis of MDOF models. Figure 4.4, shows the proposed approach for downsampling of records applied to an idealized model of a six-story building modeled in OpenSees. Figure 4.4 shows the correlation between the maximum displacement caused by original acceleration and that created by its approximation. The estimated response values are almost overlapping; error is small and in an acceptable level. In addition, for most of the cases, the new time steps are around 0.09s. Compared with the original time step 0.005s, the calculation efficiency can be improved at least 18 times.

Figure 4.4. Comparison between response of a 6-story MDOF system using original and downsampled ground motion.

Nonlinear Forty-Story Buckling Restrained Brace Frame Results Using Proposed Downsampling Method

75 NGA ground motions were run into an analytical model of a 40 story buckling restrained brace frame. The lateral load resisting system of the structure utilizes buckling restrained braces and concrete filled steel plate box-columns. The structure was designed according to the design criteria published by the Los Angeles Tall Building Design Council (LATBSDC, 2008), a code document which is intended to provide guidelines for professional engineers to implement
performance based design of tall buildings. Additional information on the design and modeling can be found in Dutta and Hamburger (2009a, 2009b, 2010), & Moehle e., al. (2009, 2011). The results were compared in terms of roof displacement. Figure 4.5 shows that the red line representing the 50th percentile (median) is approximately at 0.02 error, and 84% of error is smaller than 0.04. The reduction of time step to around 0.05s compacting the calculation time into one over tenth of original one.

![Figure 4.5](image_url)

Figure 4.5. Error in 40-story MDOF response using the proposed method for reducing ground motion sampling frequency

**Summary and conclusion**

The objective of this paper is to propose a representation of earthquake ground motion records by a relatively small number of signals, which would be efficient for primarily keeping the characteristic of ground motion and highly reducing the model analysis time. By taking advantage of FRF, we are able to foresee the characteristics of structural response in frequency domain and identify the important content (1% of maximum amplitude) to filter, downsampling and accomplish the efficient structural analysis. The FRF theory only applies to linear analysis. However, when it comes to nonlinear behavior, since most structures go through period elongation (frequency become smaller), a time step based on linear behavior should usually be short enough for calculating inelastic response. The goodness of the approximation was measured in terms of the ability to represent the response for SDOF to MDOF, from linear to nonlinear structural systems reasonably well, representing all kinds of structures. The correlation between these quantities obtained from the actual signal and from its approximation was computed for 3122 ground motions from the NGA database. The results show very high correlation for the whole displacement response. In the linear domain, the cross correlation coefficients are very stable (>90%) In nonlinear domain, the response is more sensitive to the change of input acceleration. The coefficient of correlation for the whole displacement response is smaller, but it still exceeds 0.95 with 84% probability. Another parameter, that researchers and engineers are interested in, is the maximum displacement. Results show that the most errors are <10%, especially in the high period range.
References


