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ACCIDENTAL TORSION IN NONLINEAR RESPONSE HISTORY ANALYSIS

J. A. Jarrett[1], R. B. Zimmerman[2], and F. A. Charney[3]

ABSTRACT

Accidental torsion is not required in most of the recommended provisions for nonlinear response history analyses (NRHA), including those of FEMA P-695, FEMA P-58, and the PEER Tall Building Initiative. Linear procedures for assessing accidental torsion exist in ASCE 7 Chapter 12 and ASCE 41 Chapter 3 where the center of mass is shifted 5 percent of the diaphragm dimension in each direction independently. However, studies have shown that accidental torsion can have a significant effect on the inelastic behavior of structures, and the traditional elastic methods of assessing accidental torsion may not sufficiently capture the potential inelastic torsion behavior. These effects can be especially important for structures with minimal inherent elastic torsion. This research uses a torsionally-regular five-story steel structure to investigate the effects of accidental torsion in NRHA. Several methods to assess accidental torsion are considered in the research, one of which is to apply the linear procedures that shift the center of mass by 5 percent of each diaphragm dimension to NRHA. Accidental torsion from random modifications in strength and stiffness are also investigated. These analyses show that accidental torsion can have a significant effect on the inelastic behavior of this structure, particularly for shifts in mass; therefore, the inclusion of accidental torsion within provisions for NRHA could be warranted. However, the four shifts in mass significantly increase the computational demand, so this work also investigates ways to minimize the required number of shifts by using either the calculated location of the center of rigidity or nonlinear pushover analyses. This paper provides suggested code language for a potential NRHA provision that includes an exception to reduce the number of mass shifts, as applicable.

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Accidental torsion is not required in most of the recommended provisions for nonlinear response history analyses (NRHA), including those of FEMA P-695, FEMA P-58, and the PEER Tall Building Initiative. Linear procedures for assessing accidental torsion exist in ASCE 7 Chapter 12 and ASCE 41 Chapter 3 where the center of mass is shifted 5- percent of the diaphragm dimension in each direction independently. However, studies have shown that accidental torsion can have a significant effect on the inelastic behavior of structures, and the traditional elastic methods of assessing accidental torsion may not sufficiently capture the potential inelastic torsion behavior. These effects can be especially important for structures with minimal inherent elastic torsion. This research uses a torsionally-regular five-story steel structure to investigate the effects of accidental torsion in NRHA. Several methods to assess accidental torsion are considered in the research, one of which is to apply the linear procedures that shift the center of mass by 5 percent of each diaphragm dimension to NRHA. Accidental torsion from random modifications in strength and stiffness are also investigated. These analyses show that accidental torsion can have a significant effect on the inelastic behavior of this structure, particularly for shifts in mass; therefore, the inclusion of accidental torsion within provisions for NRHA could be warranted. However, the four shifts in mass significantly increase the computational demand, so this work also investigates ways to minimize the required number of shifts by using either the calculated location of the center of rigidity or nonlinear pushover analyses. This paper provides suggested code language for a potential NRHA provision that includes an exception to reduce the number of mass shifts, as applicable.

Introduction

Under seismic loads, structures can be subjected to effects from two types of torsional influences: inherent torsion and accidental torsion. Inherent torsion includes the effects of the difference in location of the center of rigidity from the center of mass, while accidental torsion includes unaccounted effects from phenomena such as non-uniform ground motion input, variations in floor masses over the life of a structure, and strength and stiffness degradation during nonlinear response of lateral force-resisting elements. Accidental torsion is generally not included in performance-based earthquake engineering provisions, including the PEER Tall Building Initiative [1], the FEMA P-695 procedure [2], and the FEMA P-58 procedure [3].

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Typically, accidental torsion is only included in elastic procedures, such as ASCE 7 Chapter 12 [4] and ASCE 41 Chapter 3 [5], where the center of mass is shifted by 5 percent of the diaphragm dimension in each direction independently. The 2009 NEHRP provisions, in the resource paper on seismic response history analysis, recommend that if accidental torsion is included in the strength design of the nonlinear structural elements (i.e. ASCE 7 Chapter 12 requirements), it does not need to be included in the nonlinear response history analysis [6]. The effect of accidental torsion is investigated in this research to determine if there is a necessity for its inclusion within nonlinear response history analyses (NRHA) or if the current state of provisions for NRHA is correct in excluding it.

Studies have shown that the uncertainty in the location of the center of mass is normally the largest influence to accidental torsion [7]. This explains why the 5 percent offset procedure is typically used, since it relates to the distribution of mass on each floor. The practical concern for response history analysis is that the four shifts in mass location quadruples the number of analyses performed, since the structure must be analyzed with four shifted center of mass locations, instead of the one computed center of mass location. Another concern with this method is that while the uncertainty in the center of mass tends to contribute the most to accidental torsion, it is not the only source of accidental torsion, particularly for elastically torsionally-regular structures. Damage during earthquakes such as experienced by the Clarendon Tower during the Canterbury Earthquake Sequence suggests that accidental torsion due to asymmetric changes in strength or stiffness can be significantly important even for structures with that are elastically torsionally-regular [8]. These effects can be especially substantial in (originally) perfectly symmetric structures, and the method of shifting the center of mass in the elastic analysis based design (like those from ASCE 7 and ASCE 41) may not be sufficient to capture all the effects of accidental torsion [9]. It is important to distinguish buildings based on their sensitivity to torsional response. Some structures appear to be elastically torsionally sensitive, meaning that accidental torsion effects cause significant changes in the torsional rotation of the building when considered in the linear range of response. The classic case of this type of structure is one that is cruciform-shaped in plan. Inelastically torsionally sensitive buildings, like the previously mentioned Clarendon Tower, are those that may or may not exhibit elastic torsional sensitivity but for which accidental torsion effects significant changes in the torsional rotation of the building when responding in the inelastic range.

Recent research has shown that buildings designed with and without the traditional linear accidental torsion provisions often behave similarly when experiencing inelasticity, showing a flaw in the way accidental torsion is currently handled [10]. This was one of the first studies on accidental torsion that used a more realistic multi-story structure to investigate torsional effects, as most of the previous studies on the topic use single story structures or simplified shear-beam models. Their recommendation was to remove the accidental torsion provision in its current state, except potentially for symmetric buildings. Several additional studies have come to a similar conclusion that current code-based accidental torsion provisions do not adequately deal with nonlinear behavior. A summary of these studies can be found in a research review by De Stefano and Pintucchi [11]. While the general conclusion of these studies is that the inclusion of accidental torsion in design is unnecessary, the results could infer that linear design provisions cannot adequately capture inelastic torsional effects, especially for inelastically torsionally sensitive buildings, and that the inclusion of an accidental torsion provision should be moved to the nonlinear analysis of these structures. This idea is reinforced in a study where the linear procedure of shifting the center of mass by 5 percent was applied to nonlinear analyses to compare the linear and inelastic responses [12]. That work found that the inelastic response to accidental torsion can be significantly greater than the elastic response. This method of shifting the center of mass during nonlinear analyses was also used to determine the effect of accidental torsion on the collapse capacity of structures [13]. The effects of shifting the mass in elastically torsionally regular structures had a small effect on the collapse capacity (less than 10 percent reduction), since the collapse was controlled by lateral movements. On the other hand, shifting the mass had significant effect on the collapse capacity of elastically torsionally irregular buildings. The work presented in this paper will also investigate the effects of accidental torsion during NRHA, but specifically for the maximum considered earthquake ground motion level.

Modeling Description and Assumptions

To determine the accidental torsion effects in NRHA, a three-dimensional model representative of the lateral force-resisting system (LFRS) of an actual commercial building is used. Its LFRS consists of two lines of steel special moment-resisting frame (SMRF) in the longitudinal direction and four lines of buckling-restrained brace frame (BRBF) in the transverse direction over five stories. Reinforced concrete walls are located around the building perimeter at the lowest floor. Each floor is assumed to have a rigid diaphragm, except the first floor, where the diaphragm flexibility is explicitly modeled. The SMRFs align with the fault normal component of the ground motion, and the BRBFs align with the fault parallel component. The gravity system is not included in the model, and a leaning column is provided to capture the P-Delta effects. Foundation flexibility and soil-structure interaction were not included. It should be noted that the structure was determined to meet the elastic requirements of ASCE 7 Chapter 12 [4] as an elastically torsionally regular structure, as the maximum ratio of δ_{max} to $\delta_{average}$ is 1.02 in the SMRF direction and 1.18 in the BRBF direction, which are both less than the limiting ratio of 1.2. The building is modeled in Perform 3D [14], and the crucial nonlinear behavior is incorporated, including localized axial-moment-moment (PMM) interacting hinges for SMRF and BRBF columns, moment-rotation hinges for SMRF beam-column connections and BRBF beams outside of gusset plates, buckling-restrained brace (BRB) elements, and shear stress-shear strain material models for concrete walls. More detail on this structure and the modeling assumptions can be found elsewhere [15]. A plan view of the lateral force-resisting elements in the structure can be found in Fig. 1, including the locations of the center of mass (COM) and center of rigidity (COR), each averaged over all the stories. It is important to note that the location of these points is based on the entire structure, not just the elements shown in Fig. 1.

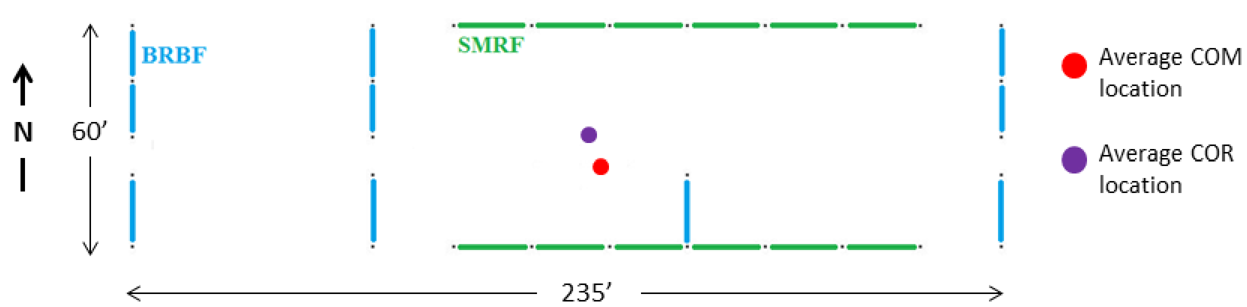


Figure 1. Schematic of the lateral force-resisting system of the example building

A target response spectrum for the maximum considered earthquake (MCE_R) is developed for the near-fault site of the structure in Berkeley, California. Due to the proximity to the Hayward Fault, the target spectrum is deterministically capped by the 80 percent lower limit of ASCE 7 Section 21.3 [4]. A suite of eleven record sets are selected to match the tectonic setting, magnitude, distance and near-fault effects controlling the seismic hazard at the site. The horizontal ground motion components of a record set are used to construct a maximum direction spectrum. The maximum direction spectra are scaled such that the mean over the suite fits the target spectrum on average and does not fall below 0.9 times the ordinate of the target spectrum in the period range of interest. More detail about the design spectrum and the ground motion selection and scaling can be found elsewhere [15]. Note that the suite average maximum direction spectrum exceeds the target spectrum in the 1.0-2.5 sec period range due to near-fault effects which are included in the selected ground motions but are not reflected in the target spectrum. Fig. 2 shows the maximum direction spectra of the scaled suite versus the target spectrum.

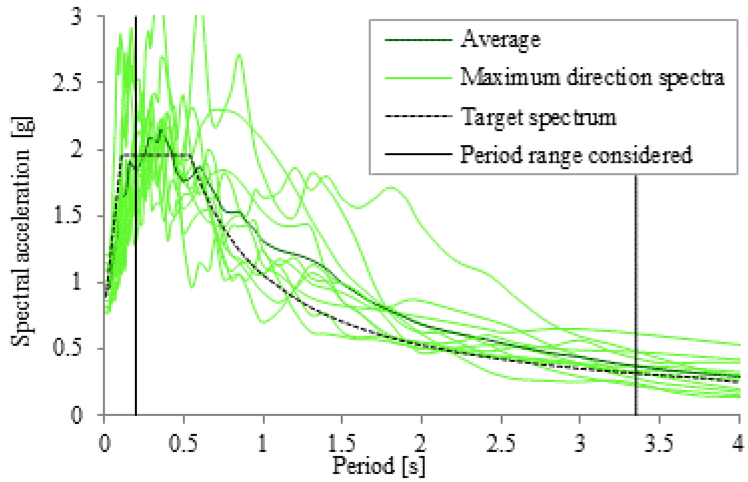


Figure 2. Maximum direction spectra versus the target spectrum for the suite of ground motions

In order to determine the effects of accidental torsion on the response during NRHA, accidental torsion is modeled in a variety of ways. Initially, the response from accidental torsion is modeled using the traditional method of shifting the center of mass. The effects of random strength and stiffness degradation are also investigated. The procedure and results of these various investigations are discussed next.

Accidental Torsion Modeled with Shifts in the Center of Mass

To investigate the effects of accidental torsion from shifts in the center of mass, eight shifts are investigated, corresponding to nine total cases including the expected center of mass location (the unaltered model). To see the most extreme effect that the torsion has on the structure, it is important to look at the displacements at the corners of the building, which are shown in Fig. 3 for the nine cases. The eight shifts can be broken into two groups: the four traditional mass offset cases and four diagonal mass offsets. The traditional cases correspond to the following eccentricities (relative to the total building dimension in the same direction): 5% East, 5% West, 5% North, and 5% South. The diagonal mass offsets are defined as Northeast, Southeast, Southwest and Northwest, where there is a 5 percent eccentricity in each of the two orthogonal directions simultaneously. For example, the Northwest eccentricity has a 5 percent eccentricity in the north direction plus a 5 percent eccentricity in the west direction. Relative to the non-diagonal shifts, the 5 percent diagonal shifts may be too conservative, but these preliminary results will provide trends between different potential mass locations. The diaphragm geometry has a rather high aspect ratio, with the dimension in the East-West far exceeding that in the North-South direction. Resulting from this, the 5 percent mass offset produces a greater shift in the North-South (BRBF) direction, and the effects in the East-West (SMRF) direction are negligible due to the small diaphragm dimension. Therefore, only the drifts in the BRBF direction are presented in Fig. 3, which shows the maximum of the four average corner drift ratios along the height of the structure.

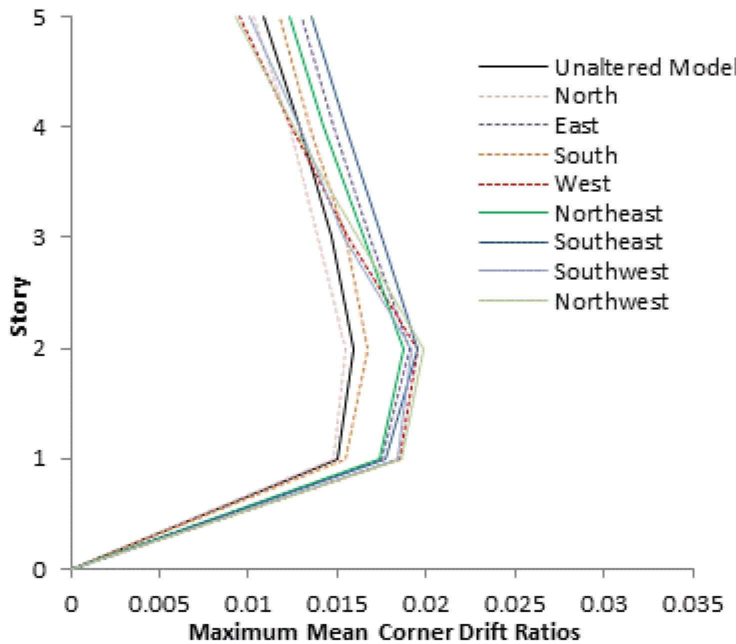


Figure 3. Corner story drift ratios computed by first averaging over all ground motions within a suite and then maximizing over each corner in the BRBF direction only.

In the BRBF direction, these mean drift ratios can increase by up to 25 percent when the 5 percent eccentricities are included. Additionally, two of the diagonal mass offsets tend to envelope the four "traditional" mass offsets, which in this case are the southeast and northwest mass offsets. Since running multiple mass offsets is computationally demanding, two procedures to determine the most crucial mass offsets are discussed in the next two sections.

Determine Which Traditional Eccentricities to Evaluate Using Pushover Data

Nonlinear pushover analysis is a method of analysis that is significantly less computationally demanding than nonlinear dynamic analyses, and it is often used to check a mathematical model for errors and to determine the distribution of inelasticity. In this procedure, nonlinear pushover analyses are additionally used to determine which traditional mass offset(s) should be analyzed using NRHA and which can be ignored. Nonlinear pushovers were performed in the North direction and the East direction at the true center of mass and at each traditional shift in the center of mass location, using the exact same mathematical model used in the dynamic analyses.

Fig. 4 shows the results from the pushover analyses in the North direction where there is a 5 percent eccentricity in the West or East direction. These figures plot the base shear against the roof drift ratio in the North direction at three locations: the approximate center of the building (Roof Y), the northeast corner (Roof NE Y) and the northwest corner (Roof NW Y). Due to the small dimension of the structure in the North-South direction, the torsional influences from the pushovers in the East direction are negligible and not shown.

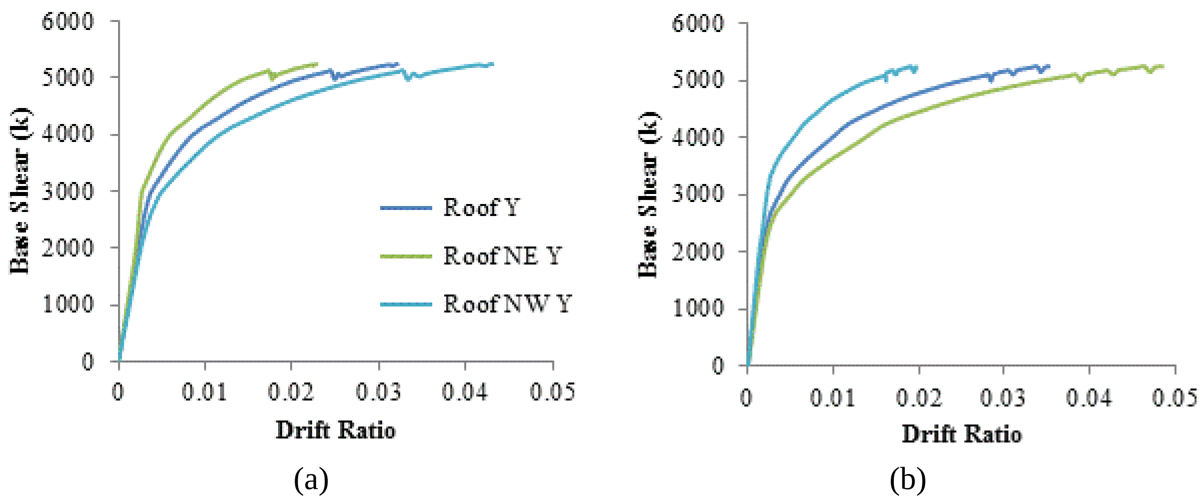
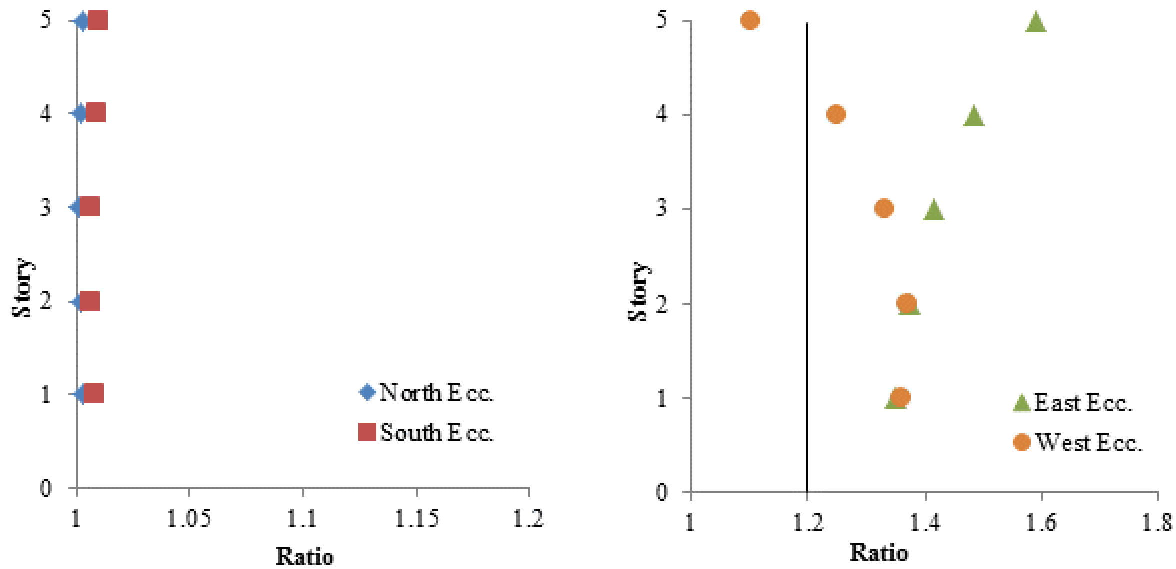


Figure 4. Pushover curves in the North direction where there is a (a) 5% eccentricity to the West and (b) 5% eccentricity to the East

When the pushover is performed at the West eccentricity, there is a 34 percent difference between the largest corner drift and the center drift at the end of the analysis. When the pushover is performed at the East eccentricity, there is a 44 percent difference between the largest corner drift and the center drift at the end of the analysis. These pushover results suggest that there could be a significant effect from accidental torsion on this structure in the nonlinear range of response, even though little is predicted for the linear range of response (recall that the structure was torsionally regular). Using the results of these pushover curves, a method to determine the necessary mass offsets could be developed. For example, the dynamic analysis associated with a given mass offset need not be performed if it is shown by nonlinear static pushover analysis using the same mass offset that the maximum corner displacement of every level is less than 1.2 times the average displacement at that same level. Fig. 5 shows these ratios of maximum corner displacement to the displacement at the COM at the target drift for the pushovers in both the North and East direction.

For this example, the target displacement used in the pushover analysis is equal to 3 percent of the roof height, computed at the center of mass. The lateral load pattern for the pushover analysis is proportional to the first mode shape in the direction of interest. Based on these results, the only accidental torsion mass offsets that would be required for this structure under the above method would be the 5 percent eccentricity in the East and West directions. As was shown in Fig. 3, the East and West eccentricity did in fact have the highest influence on the response history drift results (with the East eccentricity controlling for the upper stories and the West one controlling for the lower stories). With this method, the computational demand is significantly reduced, since only two mass offsets are required, as opposed to the traditional four.



(a) (b)

Figure 5. Ratio of maximum corner drift to center of mass drift at a COM drift ratio of 3 percent due to a pushover in the (a) East direction and (b) North direction

Alternative Method for Assessing Accidental Torsion

As Fig. 3 shows, there were two diagonal mass offsets that encompass all the traditional accidental torsion mass offsets. Assuming these diagonal mass offsets could be predicted, another method to assess accidental torsion influence would be to only run the two enveloping, diagonal mass offsets. The following procedure provides a method to determine these worst mass offsets. By calculating the true center of rigidity (COR), the worst case scenario eccentricities could be determined as the diagonal in the quadrant near and opposite to the COR. The COR for this example is determined with the method used by ETABS [16]. For every story in this structure, the true COR is located in the northwest quadrant, and the average distance between the COM and the COR is (-20.3, 92) inches or (-0.7% X, 12.8% Y). This equates to an angle of 77.6 degrees clockwise from the West axis. This location of the COR would predict that the northwest and southeast diagonals would be worst case scenarios. This matches the results shown in Fig. 3, where the two diagonal mass offsets that envelope the four “traditional” mass offsets are the southeast and northwest mass offsets.

Using the diagonal mass offsets with a 5 percent offset in each direction would still be an approximate solution, since the angle to the true location of the center of rigidity will differ from the angle to this approximate diagonal. To determine this effect, the NRHA is performed at mass locations with two different angles: at the approximate angle of 45 degrees and at the true angle of 78 degrees clockwise from the West axes. The length of these offsets is held constant as the length determined from the approximate location. Fig. 6 shows the maximum of the mean corner drift results of the response history analyses with no mass offset (Unaltered Model), at the enveloping “diagonal” mass offsets (Southeast and Northwest), and at the new mass offsets based on angle of the COR (Away from COR and Towards COR). The mass offset based on the COR provides similar results to those from the enveloping diagonal mass offset, particularly in the controlling quadrant, which is opposite to the COR. It appears, at least for this structure, that using just the approximate diagonal shift located in the quadrant opposite to the center of rigidity would sufficiently (and conservatively) predict the maximum response from the accidental torsion. Instead of four accidental torsion mass offsets, only one or two cases are needed.

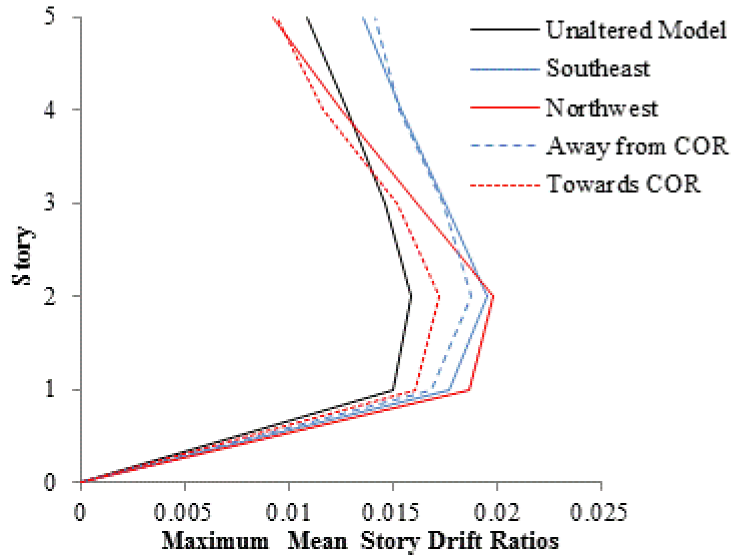
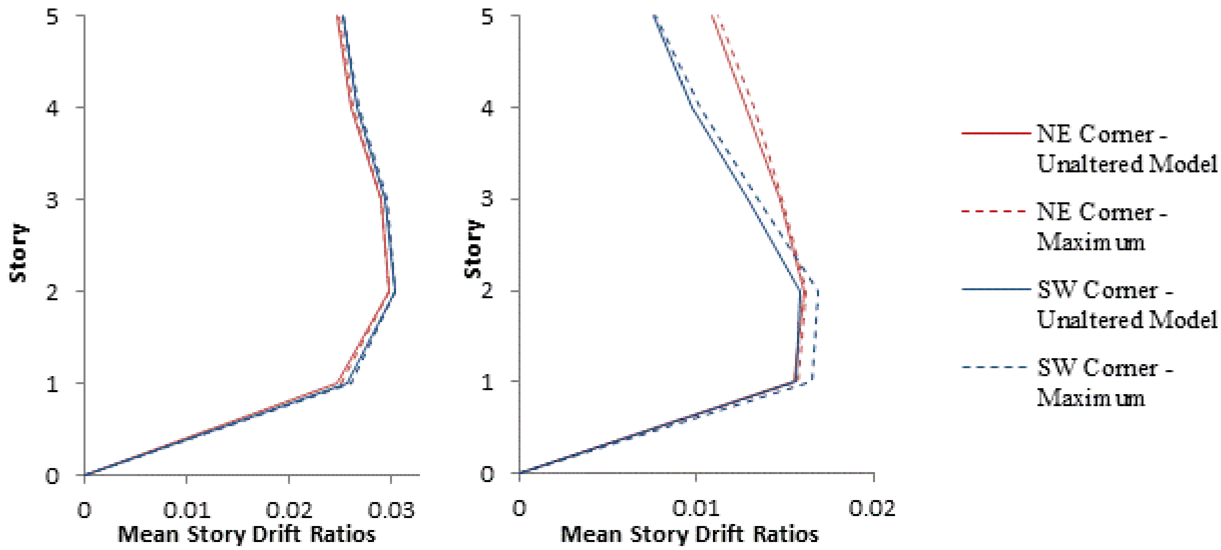


Figure 6. Corner story drift ratios computed by first averaging over all ground motions within a suite and then maximizing over each corner in the BRBF direction only.

Accidental Torsion Modeled with Random Strength and Stiffness Degradation

One concern about the traditional methods of analyzing accidental torsion by shifting mass locations is that it can only indirectly capture the torsional response from changes in strength and stiffness. As discussed before, this source is often negligible but occasionally can produce significant effects. This section investigates another approach to assessing torsional sensitivity through explicit modification of individual component strength and stiffness. Twelve different analyses were run with varying random reductions in stiffness and strength. Ten of these analyses reduce the strength or stiffness of five to ten randomly selected moment frame beams or buckling restrained braces by 10 percent. The other two analyses reduce the strength of an entire frame by 10 percent. Fig. 7 compares the suite mean story drift ratio results without any strength or stiffness reduction (Unaltered Model) to the maximum of the randomly varying strength and stiffness analyses.

There is minimal effect on this structure under random decreases in strength and stiffness. The mean change in response under the scenarios where 5 to 10 members have strength or stiffness reduction is less than 1 percent. Even when the strength of an entire BRB frame is reduced by 10 percent and the reduction is limited to one side of the structure, the maximum increase in mean drift ratio is 6.5 percent, as the results in Fig. 7 show. This procedure provides minimal insight into the behavior of this structure when compared to the method of moving the center of mass. However, this effect may have more influence on other structures, and the procedure could be useful to check if a particular structure is susceptible to this influence.



(a) (b)

Figure 7. Comparison of the suite mean story drift ratio results without any strength or stiffness reduction (Unaltered Model) to the maximum of the randomly varying strength and stiffness analyses in (a) the SMRF direction and (b) the BRBF direction

Conclusions

This work demonstrates that accidental torsion can have a significant impact on the nonlinear behavior of a structure, particularly for shifts in the center of mass for this example. This work also shows that it is possible to predict the worst cases for the location of the shifted center of mass using nonlinear pushover analysis. However, this research was performed on only one building, and it is important to expand these procedures to other structures to determine if this behavior is typical and if accidental torsion should be included in NRHA provisions. As a starting point for such work, potential example code language is shown next in italics. Again, this example code language is based on an introductory set of analyses, and more examples and engineering expertise should be used to determine its necessity and finalize the values underlined (e.g. *5 percent*) in this language.

The analysis shall consider the effect of both inherent and accidental torsion. Accidental torsion shall be included by displacing the center of mass each way (i.e. plus or minus) from its expected location by a distance equal to 5 percent of the horizontal dimension of the structure at the given floor measured parallel to the direction of mass offset. The required 5 percent displacement of the center of mass need not be applied in both orthogonal directions at the same time.

> *Exception: The dynamic analysis associated with a given center of mass offset shall not be required if the following is satisfied: It is shown by nonlinear static pushover analysis, using the same mass offset, that the maximum corner drift is less than 1.2 times the average drift at all levels of the structure. The roof target drift used in the nonlinear static analysis shall be equal to the maximum roof drift ratio permitted for the structure, and the lateral load pattern for the pushover analysis shall be proportional to the first mode shape in the direction of interest.*

Numerous previous works have highlighted the potential flaws of including the accidental torsion only in the design and linear analyses of the structure, and this work highlights the potential need for the inclusion of accidental torsion in nonlinear analyses. Assuming a structure barely satisfies the drift limits of a provision for NRHA without accidental torsion, the drifts including the nonlinear effects of accidental torsion could be dangerously large and unaccounted for. The addition of code language similar to the above could ensure the inclusion of accidental torsion in the NRHA of structures that are strongly influenced by its effect, while reducing the computational demand for structures where accidental torsion is not as critical.

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