DEVELOPMENT OF AN EFFICIENT PROCEDURE FOR PROBABILISTIC EARTHQUAKE SITE CLASS DETERMINATION

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ABSTRACT

Efficient software has been developed at the University of British Columbia (UBC) Earthquake Engineering Research Facility (EERF) to retrieve a probability distribution of subsurface shear-wave velocity (V₅) structure via inversion of low-cost surface seismic measurements. The UBC-EERF Shear-Wave Profile Probability (SWPP) software increases efficiency by streamlining the multi-step inversion process and provides outputs useful to the practicing engineer according to National Building Code of Canada (NBCC) guidelines. The field procedure involves collection of ambient vibration measurements using three-component Tromino® velocity sensors. Single-sensor recordings provide site period estimates via horizontal-to-vertical spectral ratios at each location, used to confirm uniformity of subsurface ground conditions. Multi-sensor array recordings provide dispersion estimates which are inverted for subsurface V₅ structure via the SWPP program. The SWPP software streamlines the multi-step model parameterization selection process by initiating optimization inversions for up to three different parameterizations. The most appropriate parameterization is calculated using an objective criterion that accounts for data misfit and penalizes for the number of parameters. The probabilistic inversion routine is initiated automatically for the most appropriate parameterization. Final SWPP outputs include: (1) the V₅-depth profile probability distribution, demonstrating which portions of the retrieved V₅-depth structure are well resolved and should be relied upon for seismic design, and (2) the mean (and one standard deviation) V₅₃₀ estimate and probability of associating NBCC site classification. Two case studies at high-priority schools performed as part of the British Columbia school seismic retrofit program are presented. Overall the UBC-EERF procedure described here provides information essential for earthquake site response analysis at minimal cost.

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Development of an efficient procedure for probabilistic earthquake site class determination

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ABSTRACT

Efficient software has been developed at the University of British Columbia (UBC) Earthquake Engineering Research Facility (EERF) to retrieve a probability distribution of subsurface shear-wave velocity ($V_S$) structure via inversion of low-cost surface seismic measurements. The UBC-EERF Shear-Wave Profile Probability (SWPP) software increases efficiency by streamlining the multi-step inversion process and provides outputs useful to the practicing engineer according to National Building Code of Canada (NBCC) guidelines. The field procedure involves collection of ambient vibration measurements using three-component Tromino® velocity sensors. Single-sensor recordings provide site period estimates via horizontal-to-vertical spectral ratios at each location, used to confirm uniformity of subsurface ground conditions. Multi-sensor array recordings provide dispersion estimates which are inverted for subsurface $V_S$ structure via the SWPP program. The SWPP software streamlines the multi-step model parameterization selection process by initiating optimization inversions for up to three different parameterizations. The most appropriate parameterization is calculated using an objective criterion that accounts for data misfit and penalizes for the number of parameters. The probabilistic inversion routine is initiated automatically for the most appropriate parameterization. Final SWPP outputs include: (1) the $V_S$-depth profile probability distribution, demonstrating which portions of the retrieved $V_S$-depth structure are well resolved and should be relied upon for seismic design, and (2) the mean (and one standard deviation) $V_{530}$ estimate and probability of associating NBCC site classification. Two case studies at high-priority schools performed as part of the British Columbia school seismic retrofit program are presented. Overall the UBC-EERF procedure described here provides information essential for earthquake site response analysis at minimal cost.

Introduction

An efficient and minimal-cost technique to predict earthquake site response according to current probabilistic seismic hazard analysis methodology and building code guidelines is of great interest to the University of British Columbia (UBC) Earthquake Engineering Research Facility (EERF). Inversion of surface wave dispersion curves is a well-known non-unique problem; many layered earth models may acceptably predict (fit) the measured dispersion data. Hence, an inversion process that uses low-cost surface seismic data to capture layered earth model uncertainty is a logical solution.

A non-linear Bayesian inversion routine that draws models proportional to their

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probability, combined with rigorous estimation of data error statistics and an appropriate model parameterization, was developed by Molnar et al. [1] to provide quantitative uncertainty estimates of earth model parameters. A Bayesian inversion approach formulates the inverse problem in terms of the posterior probability density (PPD) of the model parameters, which are considered random variables constrained by data and prior information. The solution is typically quantified in terms of properties of the PPD that represent optimal parameter estimates, parameter uncertainties, and parameter inter-relationships. Computing these properties requires optimizing and integrating the PPD, which must be carried out numerically for nonlinear problems. The most probable model is computed using an adaptive hybrid optimization algorithm. Parameter uncertainties and inter-relationships are computed using Markov-chain Monte Carlo methods which provide an asymptotically unbiased sample from the PPD. This Bayesian inversion approach involves multiple consecutive steps, each requiring user decision-making and interaction. For example, the type of layered earth model parameterization to use is often unknown; many different earth models may adequately fit the dispersion data. Model parameter estimates will be under-fit (biased) or over-fit (extra structure) if too few or too many parameters are used, respectively. Molnar et al. [1] use the objective Bayesian information criterion, which favors low data misfit while penalizing for extraneous parameters, to determine the most appropriate model parameterization.

The multi-step Bayesian inversion process of Molnar et al. [1] has been streamlined and converted into the UBC-EERF Shear-Wave Profile Probability (SWPP) software program to achieve probabilistic earthquake site classification according to the National Building Code of Canada (NBCC). This paper documents implementation and design of the SWPP software to provide outputs useful to the practicing geotechnical engineer. Two case studies of high-priority schools as part of the British Columbia school seismic retrofit program are presented. Results of the UBC-EERF SWPP procedure are validated at three deep borehole locations on Vancouver Island, British Columbia, Canada [2].

**UBC EERF SWPP Program**

The UBC-EERF SWPP program determines the subsurface $V_S$-depth profile probability distribution via Bayesian inversion of fundamental-mode Rayleigh wave dispersion data for use in determination of site-specific NBCC response spectra. In particular, the SWPP program also provides the mean and one standard deviation of the $V_{S30}$ estimate, and probability of associating NBCC site classification, as required currently by the NBCC for assignment of period-dependent seismic amplification factors.

The four main steps of the SWPP program (Fig. 1) are: (1) provide input data, (2) perform model parameterization study, (3) perform probabilistic (Bayesian) inversion, and (4) interpret results according to NBCC guidelines. Steps 1 and 2 require user input (darker-shade boxes), whereas steps 3 and 4 complete automatically (lighter-shade boxes). If the user does not agree with the automatic solution of step 3, the user stops the current program run and re-starts the program with different model parameter settings in step 2 (arrows depict iterative process).
Step 1: Input Dispersion Data
The SWPP program begins with the setting of the working directory and selection of input dispersion data (Fig. 2 left panel). Dispersion data may be derived by either active (e.g. hammer impact) or passive (e.g. ambient vibration) source surface wave testing.

Step 2: Model Parameterization Study
The SWPP program streamlines model parameterization selection by automatically initiating three parameterized cases (Fig. 2 right panel): two uniform gradient layers, a powerlaw gradient layer, and a linear gradient layer, overlying a homogenous half-space. Each layer is characterized by four elastic parameters: \( V_S \), layer thickness (h), compressional-wave velocity (\( V_P \)), and density (\( \rho \)). For all parameterizations, an initial value must be provided that exists between the minimum and maximum limits of the uniform prior distribution. All model parameter values are adjustable. Prior limit bounds are set wide, while encompassing realistic values, to allow the dispersion data (not the prior) to primarily determine the inverse solution. In the case of powerlaw and linear gradient layers, the \( V_S \), \( V_P \), and \( \rho \) parameter gradients are defined by prior distributions at the top and base of the gradient layer; the full layer thickness is partitioned logarithmically into a specified number of sublayers. Data uncertainties are estimated via residual analysis. The maximum-likelihood approach requires an assumption of the probability distribution for the residuals; standard assumptions of uncorrelated Gaussian-distributed data errors are applied here. Data errors are assumed to be the same for either: (1) all data points [default setting], or (2) for up to 5 subsets of data points. Option 2 is to be used when dispersion data is non-continuous and different data errors are assumed for each discrete subset of data points. For each parameterization, an optimization inversion is performed and the BIC value is calculated. The process is streamlined by automatically performing each optimization inversion for the enabled model parameterizations. The inversions are currently run consecutively. The GUI window of each step is removed as the next GUI window appears. The GUI window of step 2 cannot be closed by the user until all optimization inversions are complete.
Step 3: Automated Bayesian Inversion

Upon completion of each optimization inversion for the enabled model parameterization cases selected in step 2, the GUI window of step 3 automatically appears (Fig. 3 left panel). Each optimal inverse solution is plotted automatically for the user in terms of visual goodness-of-fit criteria: comparison of measured and predicted phase velocities with maximum-likelihood standard deviation estimates indicated as error bars (Fig. 3 top plot), and the difference between measured and predicted (residual) values (Fig. 3 middle plot). The optimal model solution is converted into a \( V_S \) depth profile and plotted (Fig. 3 bottom plot). The objective Bayesian Information Criterion (BIC) is automatically calculated for each optimal model solution. The BIC accounts for model misfit while penalizing for the number of parameters. The inverse solution with the minimum BIC value is automatically selected as the most appropriate parameterization for the subsequent Bayesian inversion. For the example in Fig. 3, the minimum BIC solution is the two-uniform layer parameterization. The results plotted in Fig. 3 are automatically saved in the user-selected working directory as an image file.

While the user examines the results of step 3 (Fig. 3 left panel), the SWPP program continues automatically. The model parameter settings of the inverse solution with the lowest BIC value are copied and used for the automatic initiation of the probabilistic inversion routine. The probabilistic inversion routine performs Markov-chain Monte Carlo sampling with an efficient implementation of Metropolis-Hastings sampling. The total maximum number of models to sample and an appropriate convergence value for the probabilistic inversion is set here to 100,000 and 0.05, respectively. To not influence convergence prematurely, the first 15,000 models (maximum) are not saved, a stage commonly referred to as ‘burn-in’. Otherwise, the probabilistic inversion solutions are saved to two ASCII text files which contain all model parameters and \( V_S \)-depth profile parameters (\( h \) and \( V_S \) only). The purpose of displaying the GUI window of step 3 (Fig. 3) is for the user to assess the adequacy of the inverse solutions. The user must re-start the program with different model parameterizations and/or different parameter prior bounds in step 2 until such time that the BIC selected model is the preferred solution for automatic initiation of step 4.
Figure 3. Screen snapshots of UBC-EERF SWPP program step 3 (left panel) and 4 (right panel).

**Step 4: Useful outputs according to NBCC guidelines**

During the probabilistic inversion, models are written to an ASCII output file. Upon completion, the probabilistic inverse solution is plotted automatically for the user (Fig. 3 right panel) in terms of the 95% and 66% highest probability density (HPD) credibility interval $V_S$ profile probability distribution. Credibility intervals are determined from binning each of the $\leq 100,000$ models of the PPD onto a fine $V_S$-depth grid from which the probability of all profiles is integrated from this gridded $V_S$-depth probability distribution. This result displays to the end user the $V_S$ depth structure, including uncertainties, as resolved by the dispersion data.

The current site response predictor of the NBCC is based primarily on the average $V_S$ of the upper 30 m. The SWPP program automatically calculates the probability distribution of the average $V_S$ at 30-m depth and reports the average and one standard deviation $V_{S30}$ estimate in an ASCII text file. The probabilities of associating NBCC site classes (A-E) based on the $V_{S30}$ probability distribution are also reported in this output text file. The end user may then apply the appropriate period-based amplification factors to the uniform hazard spectra for engineering use.
Example Case Studies

Each British Columbia (BC) school selected for seismic retrofit requires assessment of site-specific ground conditions for NBCC site classification, i.e. $V_{S30}$. The UBC-EERF surface seismic testing approach and SWPP program is a rapid and low-cost approach to provide the required site-specific $V_{S30}$ estimate with quantitative uncertainty estimates. Case studies of $V_{S30}$ assessment performed by UBC-EERF at two high-priority BC schools are presented here. School site A is located ~30 km south of Vancouver (Fig. 4). The school property bounds two surficial geological units: $\leq 5$ m scattered sand deposits and $\leq 25$ m glacial till, respectively, overlying Tertiary bedrock [3]. These geologic units are assigned to NBCC site class C (Fig. 4) [4]. School site B is located ~60 km southeast of Vancouver. The school property occurs within mapped surficial thick marine silt and clay upland deposits overlying Tertiary bedrock [3], assigned to NBCC site class D [4]. The regional amplification hazard map of Greater Vancouver provides a general approximation of the site conditions but is not applicable for site-specific purposes.

The UBC-EERF surface seismic testing procedure [2] performed at these two high-priority BC schools involves: (1) single-sensor ambient vibration recordings at select locations to provide peak frequency (site period) estimates from horizontal-to-vertical spectral ratios, and (2) simultaneous multi-sensor ambient vibration recordings of varying radii to provide dispersion estimates for inversion with the SWPP program to obtain a probability distribution of the subsurface $V_S$ structure. This two-part field procedure is designed to assess consistency of subsurface ground conditions beneath the arrays as well as the entire school property to provide confidence that the structure determined beneath the arrays is applicable for earthquake site response characterization of the entire school site. Ambient vibration recordings are performed...
using tri-axial Tromino® seismic sensors; standalone, portable instruments with internal GPS and low power consumption (tromino.eu). Single-sensor recordings are performed sporadically around the entire school property and each recording is ~15-20 minute duration. Simultaneous multi-sensor array recordings are performed at a given radius for ~20-30 minute duration in an open area within the school property; the array radius is varied several times.

Figure 5a shows the average H/V ratios for the smallest and largest radii array locations (blue and red circles in Fig. 5b). Peak frequencies are ~0.9 Hz (site period of ~1.1 s) at these array locations and range between 0.7-0.9 Hz (1.1-1.4 s) at 15 other locations around the school property (not shown). Confidence is provided from the observed consistent peak frequencies that structure determined beneath the array is applicable to characterize ground conditions of the entire school site. Figure 5c (left panel) shows the Rayleigh-wave phase velocity estimates extracted from the multi-sensor array recordings (locations shown in Fig. 5b) used for input to the SWPP program. Phase velocities increase rapidly from < 200 m/s at high frequency (near surface) to ~400-500 m/s at low frequency (at depth). The SWPP program determines that the most appropriate model parameterization (minimum BIC model) consists of two uniform layers overlying an elastic half-space. Figure 5c (right panel) shows the resulting Vs profile probability distribution; a well-resolved shallow (~4.5 m) layer of low-velocity (~160 m/s) material overlies stiff (~520 m/s) material. The depth of the stiff material layer is not resolved, as expected by the plateau of phase velocities ~400-500 m/s at < 10 Hz and the relatively low site period (1.1-1.4 s). The upper 30-m Vs structure is well resolved and applicable for NBCC site classification. The SWPP program determines the mean Vs30 estimate and its one standard deviation uncertainty is 409 m/s and 38 m/s, respectively, corresponding to NBCC site class C (360 m/s < Vs30 < 760 m/s).

For school site B, the average H/V ratios for the smallest and largest radii array locations (blue and red circles in Fig. 6b) display consistent peak frequencies of ~0.9 Hz (~1.1 s) and at 6 other locations around the school property (not shown). The extracted dispersion data (Fig. 6c left panel) varies from ~460 m/s at 2.5 Hz to ~280 m/s at 16 Hz. The SWPP program determines that the most appropriate model parameterization (minimum BIC model) consists of two uniform layers overlying an elastic half-space. The resulting Vs profile probability distribution is shown in Fig. 6c (right panel): a well-resolved shallow (< 20 m) layer of ~270 m/s velocity overlies stiff (~420-550 m/s) material. Velocity at surface is slightly higher than at 10-20 m depth, related to an increase in phase velocities > 11 Hz and likely represents an over-consolidated or desiccated crust of the upland glaciomarine sediment. The depth of the stiff material layer is not resolved; dispersion data < 4 Hz is fit on average and the observed low site period (~1 s) suggests a relatively deep site. Overall the dispersion data exhibits more structure (kinks) than is able to be resolved by the inversion (Fig. 6c left panel). The SWPP program determines the mean Vs30 estimate and its one standard deviation uncertainty is 295 m/s and 13 m/s, respectively, corresponding to site class D (360 m/s < Vs30 < 760 m/s).
Figure 5. UBC-EERF surface seismic testing results for school site A. (a) Peak frequencies observed at selected array sites. (b) Locations of array sensors shown by filled circles colored according to simultaneous recording. (c) Results of the SWPP program: left panel shows dispersion data (filled circles) with maximum-likelihood standard deviation estimates indicated as error bars and predicted dispersion curve for the optimal inverse solution (solid line), right panel shows mean $V_s$ model and 95% and 66% HPD credibility intervals.
Conclusions

The UBC EERF has implemented the use of low-cost surface seismic testing to obtain Rayleigh wave dispersion data for probabilistic (Bayesian) inversion. The SWPP software was developed to streamline and automate the multi-step inversion process, thereby increasing efficiency. The final outputs of the SWPP software are designed to be useful to the practicing engineer: probabilistic estimates of $V_s$-depth profiles useful for site-specific response spectra, and $V_{S30}$ required for NBCC site classification and assignment of period-dependent amplification factors. The UBC-EERF surface seismic testing procedure and SWPP software is being used to perform probabilistic site classification at high-priority schools as part of the BC school seismic retrofit program. Case studies performed at two high-priority BC schools determine well-resolved $V_s$-depth profiles to at least 50-m depth (bedrock velocities are not measured). An estimate of $V_{S30}$
and its uncertainty is achieved in both cases, and probability of associating NBCC site classification is consistent with expectations based on regional surficial geology. The UBC-EERF procedure and SWPP software are validated by comparison with downhole $V_s$ measurements at three deep borehole locations in BC [2]. Overall, the development of an efficient procedure for probabilistic earthquake site class determination by UBC-EERF is a success.

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