SHAKING TABLE DEMONSTRATION OF DYNAMIC RESPONSE OF BASE-ISOLATED BUILDINGS

***** *Instructor Manual* *****

A PROJECT DEVELOPED FOR THE
UNIVERSITY CONSORTIUM ON INSTRUCTIONAL SHAKE TABLES

http://wusceel.cive.wustl.edu/ucist/

Required Equipment:

- Instructional Shake Table
- Power Supply (Universal Power Module)
- Data Acquisition and Control System (MultiQ Board and Computer)
- Software
- Measurement Sensors (Three Accelerometers)
- Test Structure (One-Story Building Model)
- Seismic Isolation Systems

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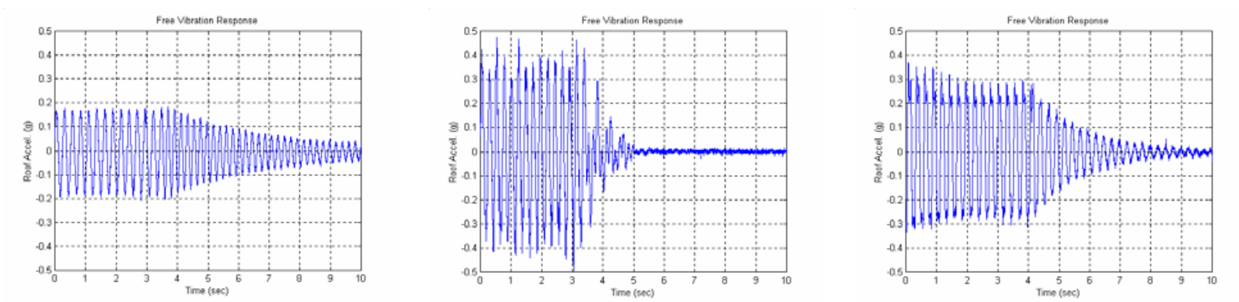
Note: Section numbers, section titles, and equation numbers are with reference to the Student Manual.

4.0 Procedures for Conducting Experimental Tests

4.1 System Identification: Free-Vibration Tests

Free-vibration response plots for the fixed-base and base-isolated structures are shown in Figure 1.



Fixed-Base **Sliding Isolation** **Elastomeric Isolation**

Figure 1 Response of fixed-base and base-isolated structures from free-vibration testing.

Recall the five-step procedure from Section 2.4.1 for system identification from free vibration test data:

1. Estimate the mass, m, of the structure.
2. Measure the undamped natural period, T_n, from the free vibration response.
3. From the undamped natural period and mass, evaluate the stiffness using Eq. (16) and (17).
4. Evaluate the damping ratio, ξ, using Eq. (23).
5. Having the mass, stiffness and damping ratio, determine the damping coefficient from Eq. (15).

Each step is carried out below for the fixed-base and base-isolated structures.

Fixed-Base Structure *(left-hand plot of Figure 1)*

Step 1: The weight of the structure is 3.41 lb and thus the mass is 0.008825 lb-s^2/in.

Step 2: $T_n = 0.211$ sec; $f_n = 1/T_n = 4.74$ Hz; $\omega_n = 2\pi f_n = 29.8$ rad/sec (natural period determined from average value over 21 cycles of free vibration response)

Step 3: $k = m\omega_n^2 = 0.008825*(29.8)^2 = 7.84$ lb/in

Step 4: $\xi = \frac{1}{2j\pi}\ln\left(\frac{u(t)}{u(t+jT_d)}\right) = \frac{1}{2(21)\pi}\ln\left(\frac{0.1556}{0.0444}\right) = 0.009504 = 0.95\% \approx 1.0\%$

(damping ratio determined using 21 cycles of free vibration response)

Step 5: $c = 2m\omega_n\xi = 2*0.008825*29.8*0.009504 = 0.00500$ lb-s/in

Base-Isolated Structure with Sliding Isolation System *(center plot of Figure 1)*
The free vibration response of the base-isolated structure with the sliding isolation system clearly does not exhibit a decaying exponential form. The primary reason for this is that the sliding isolation system responds in a highly nonlinear fashion due to the stick-slip behavior at the sliding interface. Thus, a natural period and equivalent viscous damping ratio can not be extracted from this data. *Note: It is not expected that the students will recognize this issue before testing is conducted. Consider letting them discover it and propose reasons for it.*

Base-Isolated Structure with Elastomeric Isolation System *(right-hand plot of Figure 1)*
The free vibration response of the base-isolated structure with the elastomeric isolation system exhibits a simple harmonic decaying exponential form, suggesting that the structure is vibrating in a single mode, which is assumed to be the fundamental mode.

Step 1: The weight of the structure is 6.82 lb and thus the mass is 0.017650 lb-s^2/in.

Step 2: $T_n = 0.257$ sec; $f_n = 1/T_n = 3.89$ Hz; $\omega_n = 2\pi f_n = 24.4$ rad/sec

Step 3: $k = m\omega_n^2 = 0.017650*(24.4)^2 = 10.51$ lb/in

Step 4: $\xi = \frac{1}{2j\pi}\ln\left(\frac{u(t)}{u(t+jT_d)}\right) = \frac{1}{2(12)\pi}\ln\left(\frac{0.2133}{0.0389}\right) = 0.02257 \approx 2.3\%$

Step 5: $c = 2m\omega_n\xi = 2*0.017650*24.4*0.02257 = 0.01944$ lb-s/in

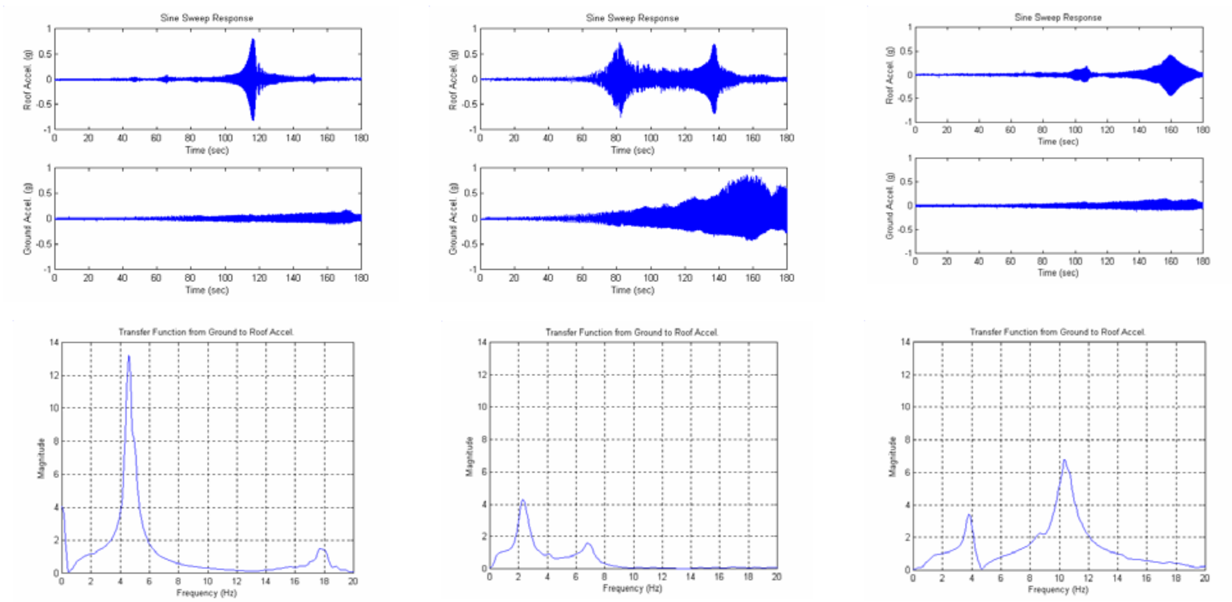
A summary of the results from free-vibration system identification testing is provided in Table 1.

Table 1 Summary of results from free-vibration system identification testing.

Configuration	**m (lb-s^2/in)**	**c (lb-s/in)**	**k (lb/in)**	**T_n (sec)**	**ξ (%)**
Fixed	0.008825	0.00500	7.84	0.211	1.0
Sliding	NA	NA	NA	NA	NA
Elastomeric	0.017650	0.01944	10.51	0.257	2.3

Note: NA = Not applicable due to strongly nonlinear response.

4.2 System Identification: Sine Sweep Tests
Acceleration response plots for the roof and foundation and the associated transfer functions are shown in Figure 2 for the fixed-base and base-isolated structures.



Fixed-Base **Sliding Isolation** **Elastomeric Isolation**

Figure 2 Roof and foundation accelerations and associated transfer functions for sine-sweep tests.

Recall the five-step procedure from Section 2.4.2 for system identification from sine-sweep test data:

1. Estimate the mass, m, of the structure.
2. Identify ω_n from the location of the peak in the R_d curve.
3. From the natural frequency and mass, evaluate the stiffness using Eq. (16).
4. Evaluate the damping ratio, ξ, using Eq. (36).
5. Having the mass, stiffness and damping ratio, determine the damping coefficient from Eq. (15).

Each step is carried out below for the fixed-base and base-isolated structures. Note that we use the acceleration transfer function, H, rather than the displacement response factor, R_d, to identify the system properties.

Fixed-Base Structure *(left-hand plot of Figure 2)*
Step 1: The weight of the structure is 3.41 lb and thus the mass is 0.008825 lb-s^2/in.
Step 2: f_n = 4.62 Hz; $T_n = 1/f_n$ = 0.217 sec; $\omega_n = 2\pi f_n$ = 29.0 rad/sec (natural cyclic frequency determined from location of lowest frequency peak)
Step 3: $k = m\omega_n^2 = 0.008825*(29.0)^2$ = 7.42 lb/in

Step 4: $\xi = \dfrac{1}{2(R_d)_{max}} \approx \dfrac{1}{2H_{max}} = \dfrac{1}{2*13.22} = 0.03782 \approx 3.8\%$

Step 5: $c = 2m\omega_n\xi = 2*0.008825*29.0*0.03782 = 0.01936$ lb-s/in

Base-Isolated Structure with Sliding Isolation System *(center plot of Figure 2)*
Although the sliding isolation system is strongly nonlinear, the transfer function reveals a dominant peak at the fundamental frequency. Thus, we can estimate the natural period of the fundamental mode.

Step 1: The weight of the structure is 6.82 lb and thus the mass is 0.017650 lb-s^2/in.

Step 2: f_n = 2.28 Hz; $T_n = 1/f_n$ = 0.438 sec; $\omega_n = 2\pi f_n$ = 14.3 rad/sec (natural cyclic frequency determined from location of lowest frequency peak)

Step 3: $k = m\omega_n^2 = 0.017650*(14.3)^2 = 3.62$ lb/in

Step 4: Damping ratio not computed since beyond scope of project.

Step 5: Damping coefficient not computed since beyond scope of project.

Base-Isolated Structure with Elastomeric Isolation System *(right-hand plot of Figure 2)*

Step 1: The weight of the structure is 6.82 lb and thus the mass is 0.017650 lb-s^2/in.

Step 2: f_n = 3.78 Hz; $T_n = 1/f_n$ = 0.265 sec; $\omega_n = 2\pi f_n$ = 23.8 rad/sec (natural cyclic frequency determined from location of lowest frequency peak)

Step 3: $k = m\omega_n^2 = 0.017650*(23.8)^2 = 9.96$ lb/in

Step 4: Damping ratio not computed since beyond scope of project.

Step 5: Damping coefficient not computed since beyond scope of project

A summary of the results from sine-sweep system identification testing is provided in Table 2.

Table 2 Summary of results from sine-sweep system identification testing.

Configuration	m (lb-s^2/in)	c (lb-s/in)	K (lb/in)	T_n (sec)	ξ (%)
Fixed	0.008825	0.01936	7.42	0.217	3.8
Sliding	0.017650	NA	3.62	0.438	NA
Elastomeric	0.017650	NA	9.96	0.265	NA

Note: NA = Not applicable since beyond scope of project.

4.3 Seismic Response Evaluation

Interstory drift and shear force response plots for the fixed-base and base-isolated structures are shown in Figures 3 and 4, respectively, for the three different earthquake motions. The peak values of each response are summarized in Table 3.

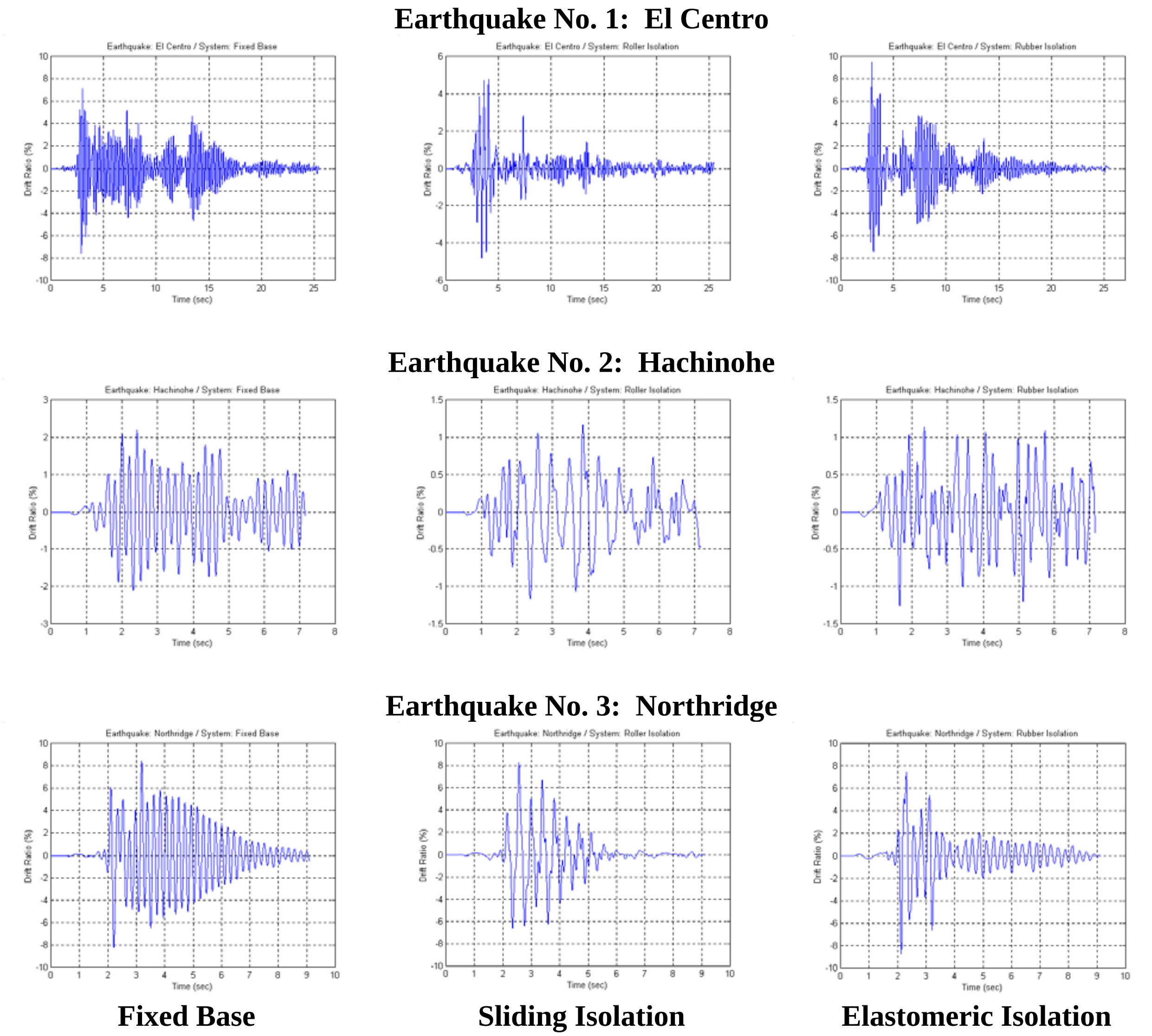
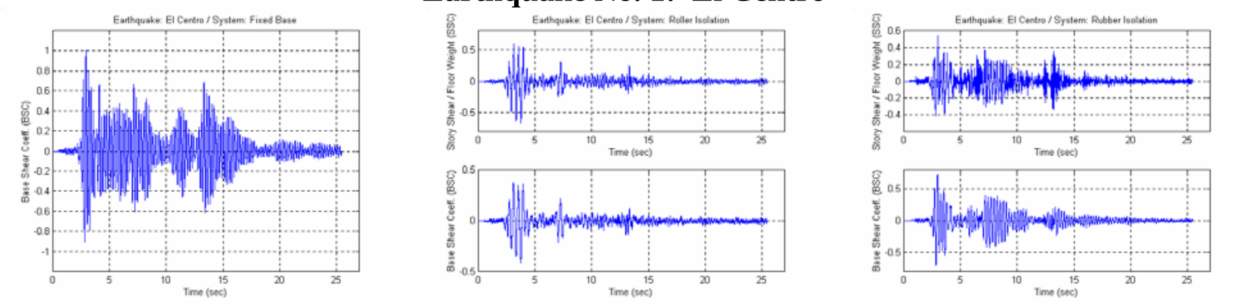
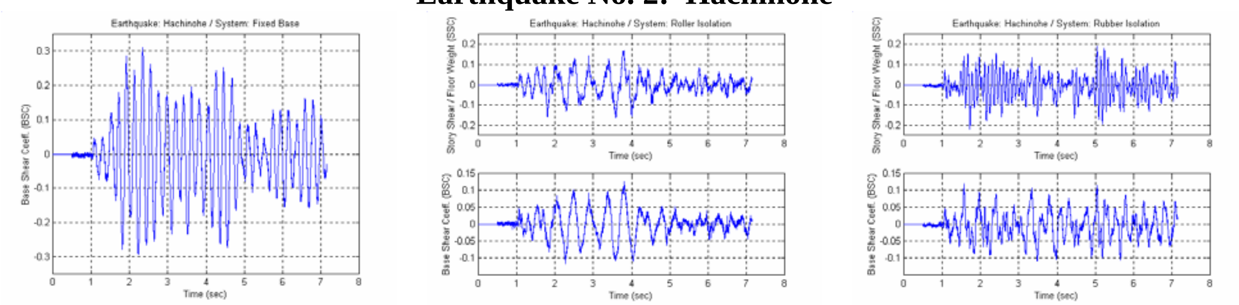


Figure 3 Interstory drift ratio responses for seismic testing.

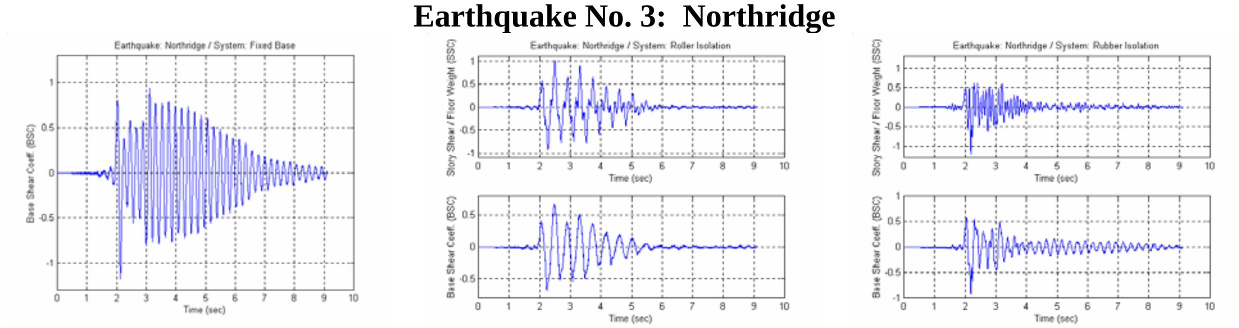
Earthquake No. 1: El Centro



Earthquake No. 2: Hachinohe



Earthquake No. 3: Northridge



Fixed Base **Sliding Isolation** **Elastomeric Isolation**

Figure 4 Shear force responses for seismic testing.

Table 3 Peak response of structures subjected to seismic testing.

Earthquake	Configuration	Drift Ratio (%)	Story Shear / Roof Weight	Base Shear / Total Weight
El Centro	Fixed	7.59	1.004	1.004
	Sliding	4.81 (-36.6%)	0.663 (-34.0%)	0.420 (-58.2%)
	Elastomeric	9.45 (+24.5%)	0.538 (-46.4%)	0.730 (-27.3%)
Hachinohe	Fixed	2.19	0.310	0.310
	Sliding	1.17 (-46.6%)	0.169 (-45.5%)	0.125 (-59.7%)
	Elastomeric	1.26 (-42.5%)	0.219 (-29.4%)	0.120 (-61.3%)
Northridge	Fixed	8.41	1.181	1.181
	Sliding	8.23 (-2.1%)	1.002 (-15.2%)	0.689 (-41.7%)
	Elastomeric	8.69 (+3.3%)	1.212 (+2.6%)	0.914 (-22.6%)

Note: Numbers in parenthesis indicate percentage change with respect to fixed-base configuration.

5.0 Questions

5.1 Questions Related to System Identification and Seismic Response Tests

1. For the free-vibration tests, is the free-vibration decay exponential for all three structural systems? In the cases where the decay is exponential, what assumption can reasonably be made about the form of damping in the structure? If the decay is not exponential, offer an explanation as to why this is the case. Comment on the system properties obtained from the free-vibration tests.

The free vibration response for the fixed-base structure clearly demonstrates the development of steady-state sinusoidal motion followed by a period of free vibration. Furthermore, the free-vibration decay is of a decaying exponential form and thus it is apparent that the assumption of linear viscous damping is reasonable for the fixed-base structure. The free vibration response of the structure with an elastomeric isolation system also decays exponentially indicating that a linear viscous damping model is appropriate. In contrast, the sliding isolation system does not exhibit an exponentially decaying free vibration response and thus a linear viscous damping model does not appear to be appropriate. The behavior of the structure with the sliding isolation system is dominated by the behavior at the sliding interface which is characterized by strongly nonlinear stick-slip behavior. Thus, it is not surprising that a linear viscous damping is not appropriate for characterizing the energy dissipation of the sliding isolation system. Finally, note that the isolated structures actually respond as two degree-of-freedom systems, although the free vibration response of the structure with the elastomeric isolation system suggests that the structure is vibrating in a single mode (i.e., only one harmonic is present). The system properties obtained from the free-vibration tests reveal that the use of an elastomeric isolation system increased the fundamental natural period and damping ratio by 21.8% and 137.5%, respectively, as compared to the fixed-base structure. The elongation of the period and the larger damping ratio indicate that the isolated structure should be effective in resisting earthquake ground motion.

2. For the sine-sweep test, how many peaks appear in the transfer functions for each structural system? What is the significance of the number of peaks? What is the significance of the location and the height of the peaks? Comment on the system properties obtained from the sine-sweep tests. Compare the results obtained from the free-vibration and sine-sweep tests.

For the fixed-base structure, the roof acceleration time-history exhibits a single global peak as the sine-sweep passes through the resonant frequency of the structure. For the two base-isolated structures, the roof acceleration exhibits two global peaks as the sine-sweep passes through the two resonant frequencies associated with the two modes of vibration.

For the fixed base structure, the transfer function exhibits a single dominant peak which occurs at the resonant frequency. Relative to the transfer functions for the base-isolated structures, the magnitude (height) of the peak is large indicating that the fixed-base structure has low inherent damping. The transfer functions for the two base-isolated structures exhibit two peaks that occur at the resonant frequencies associated with the two modes of vibration. The system properties obtained from the sine-sweep tests reveal that the use of the sliding and elastomeric isolation systems increased the fundamental natural period by 101.8% and 22.1%, respectively, as compared to the fixed-base structure. The elongation of the period indicates that the isolated structures should be effective in resisting earthquake ground motion. Note that, strictly speaking, the concept of the transfer function is only applicable to linear systems and thus the transfer function shown for the sliding isolation system is not unique. However, for an idealized sliding isolation system that incorporates a Coulomb friction sliding interface and a linear spring element, the natural frequency is unique and is precisely equal to that given by Equation (16).

A comparison of the results from the free-vibration and sine-sweep system identification tests indicates that both methods result in close estimates of the natural periods for the fixed-base structure (0.211 and 0.217 sec, respectively) and elastomeric isolation system (0.257 and 0.265 sec, respectively). The estimates of the damping ratio for the fixed-base structure are appreciably different (1.0 and 3.8%, respectively). It is likely that the damping ratio as obtained from the free vibration test is more accurate since the peak value of the transfer function is often difficult to capture experimentally, particularly for structures with low levels of damping.

3. For the seismic tests, discuss the effectiveness of the isolation systems in controlling the response. Based on your discussion, what advantages and disadvantages are associated with the use of an isolation system?

The peak seismic responses, as summarized in Table 3, indicate that both the sliding and elastomeric isolation systems were generally effective in reducing both the interstory drift response (reductions as high as 46%) and shear force response (reductions as high as 59%). However, in two cases, the seismic tests indicated that the elastomeric isolation system increased the drift ratio. This behavior is not expected for a well-designed elastomeric isolation system and suggests that alternative designs could likely be developed that would result in improvements in performance of the elastomeric isolation system.

Some advantages of a base isolation system are the expected reduction in interstory drifts, and thus a reduction in structural and non-structural damage. In addition, reductions in shear forces are associated with reduced accelerations and thus reductions in damage to the building contents. In general, a base isolation system reduces the seismic demand on the superstructure. Some of the disadvantages of a base isolation system are the need to accommodate the deformation demands at the isolation level (e.g., flexible utility connections and a building moat), the potentially more complex design process, and the potential sensitivity of isolation system performance to the point-in-time dynamic properties of the bearings (e.g, contaminants may change the sliding coefficient of friction of sliding bearings).

5.2 General Dynamic Analysis Questions

1. *What are the main differences between static and dynamic analysis?*

   In static analysis, only restoring forces are considered and the loading is not time-dependent. In dynamic analysis, in addition to restoring forces, inertia forces and damping forces are considered and the loading is time-dependent.

2. *Why do structural engineers need to understand structural dynamics?*

   Complex structural systems (e.g., base-isolated structures) may require dynamic analysis.

3. *The primary purpose of dynamic analysis is either system identification or response analysis. Are these two purposes interrelated? How?*

   Yes, these two purposes are interrelated in that response analysis requires properties obtained via system identification testing.

4. *The damped natural period of a structure is often approximated by the undamped natural period. What error is incurred by this assumption if the damping ratio of the structure is at the approximate upper bound of 10%. Based on your result, do you think the assumption is reasonable?*

   $$T_d = \frac{T_n}{\sqrt{1-\xi^2}}$$

   $$\text{Error} = \frac{T_d - T_n}{T_d} = 1 - \frac{T_n}{T_d} = 1 - \sqrt{1-\xi^2} = 1 - \sqrt{1-0.1^2} = 0.0050 = 0.5\%$$

   The error is 0.5% which is small and thus the assumption is reasonable.

5. In a sine sweep system identification test, the circular frequency at which the maximum displacement response factor occurs is often approximated as the undamped natural circular

frequency. What error is incurred by this assumption if the damping ratio of the structure is at the approximate upper bound of 10%. Based on your result, do you think the assumption is reasonable?

$$\omega_{max} = \omega_n \sqrt{1-2\xi^2}$$

$$\text{Error} = \frac{\omega_n - \omega_{max}}{\omega_{max}} = \frac{\omega_n}{\omega_{max}} - 1 = \frac{1}{\sqrt{1-2\xi^2}} - 1 = \frac{1}{\sqrt{1-2*0.1^2}} - 1 = 0.0102 = 1.0\%$$

The error is 1.0% which is small and thus the assumption is reasonable.

6. *If you are evaluating the response of a very flexible structure subjected to very fast sinusoidal loading, do you expect the displacement response to be approximately in phase, 180^o out of phase, or 90^o out of phase with respect to the applied load?*

   180^o out of phase

7. *Numerical analysis of structural systems often requires the solution of second-order differential equations. For example, the following equation of motion for earthquake loading must be solved numerically:* $m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}_g(t)$
   For numerical analysis, this equation can be solved by rewriting it as a system of two first-order differential equations via a state-space transformation. Perform this transformation. Hint: Let the state-space variables be $x_1(t)$ *and* $x_2(t)$ *where* $x_1(t) = u(t)$ *and* $x_2(t) = \dot{u}(t)$. *The derivatives of* $x_1(t)$ *and* $x_2(t)$ *are* $\dot{x}_1(t) = \dot{u}(t)$ *and* $\dot{x}_2(t) = \ddot{u}(t)$. *Now write the two first-order equations by solving the equation of motion for* $\ddot{u}(t)$ *and substituting in the state-space variables.*

$$\dot{x}_1(t) = ...$$
$$\dot{x}_2(t) = ...$$

   The above two equations will be first-order equations that are coupled in the variables $x_1(t)$ *and* $x_2(t)$. *The benefit of the transformation is that first-order equations can be solved instead of second-order equations. The cost of the transformation is that it results in twice as many equations to solve.*

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\frac{c}{m}x_2(t) - \frac{k}{m}x_1(t) - \ddot{u}_g(t)$$

8. *Bonus: Using the identified system properties for the fixed base structure, develop numerical predictions of the response of the structure when subjected to the Northridge earthquake. Compare the numerical predictions with the experimental test data.*
   Solution not provided for this question.