



*George E. Brown, Jr. Network for*  
Earthquake Engineering Simulation



## **Hybrid Simulation:**

### **A Discussion of Current Assessment Measures**



This report documents the discussions of the *Hybrid Simulation Task Force* meeting held March 27-28, 2014, in West Lafayette, Indiana.

Authored by Richard Christenson, Shirley Dyke, Jian Zhang, Gilberto Mosqueda, Cheng Chen, Narutoshi Nakata, Patrick Laplace, Wei Song, Yunbyeong Chae, Gemez A. Marshall, Ge Ou, Carlos Andres Riascos Gonzales, Cheng Song.

**Dated:** August 2014

**Sponsor:** US National Science Foundation (NSF) under Award Number CMMI-0927178.  
Sample data used within this report is published at [nees.org](http://nees.org).

## **Abstract**

*The purpose of this document is to provide a summary of the current state of assessment measures that may be implemented by users interested in evaluating the fidelity of a hybrid simulation. After reading this document, we anticipate that hybrid simulation users will have a better understanding of recent advances in hybrid simulation, consider a proposed definition of a successful hybrid simulation, and access available resources for implementation of hybrid simulation assessment measures. This document provides a summary of various current assessment measures for hybrid simulation users to evaluate their tests, and demonstrates implementation of these using results from sample data pulled from actual hybrid simulations available in the NEEShub. A suggested protocol is included for users to plan a hybrid simulation and isolate sources of uncertainty to reduce errors during testing. Interested readers are also directed to the Hybrid Simulation Primer and Dictionary [8].*



## Table of Contents

<b>Abstract</b> .....	<b>2</b>
<b>1 Introduction</b> .....	<b>4</b>
<b>2 Sources of error/uncertainties</b> .....	<b>6</b>
2.1 Assumptions used in numerical models .....	7
2.2 Numerical errors/incompatibilities related to the equation of motion.....	7
2.3 Experimental errors related to random and systematic experimental errors.....	8
<b>3 Assessment Measures</b> .....	<b>8</b>
3.1 Local Response Assessment.....	9
3.2 System-level, or Global Response Assessment .....	15
<b>4 Example Analysis</b> .....	<b>20</b>
4.1 Performance-Based Design and Real-Time Large-Scale Testing to Enable Implementation of Advanced Damping Systems (Lehigh University) .....	20
4.2 Semiactive Control of Nonlinear Structures (University of Colorado at Boulder) .....	21
<b>5 Recommendations and Request to the Community</b> .....	<b>22</b>
<i>Database of Hybrid Simulation Assessment Measures</i> .....	22
<i>How can NEEShub better meet the needs of the HS community?</i> .....	22
<b>6 Summary</b> .....	<b>24</b>
<b>References</b> .....	<b>24</b>
<b>Appendix 1. Nomenclature</b> .....	<b>26</b>
<b>Appendix 2. Hybrid Simulation Protocol</b> .....	<b>28</b>
<b>A2.1 Pretest stage</b> .....	<b>28</b>
<b>A2.2 HS stage</b> .....	<b>31</b>
<b>A2.3 Processing stage</b> .....	<b>31</b>

## 1 Introduction

Seismic evaluation of structural systems has traditionally been explored using either experimental methods or analytical models. Dynamic testing using large scale shake tables is generally viewed as the most realistic method for such evaluation. However, this approach requires access to a large shake table, which is not readily available in most structural labs. Furthermore, issues of scale, equipment capacity and availability of research funding limit large-scale shake table testing of complete structures. Analytical models, on the other hand, are limited to solving specific types of problems and in many cases fail to capture the complex behaviors or failure modes at system-level. Combining both experimental and analytical components in a single simulation (see Fig. 1), while taking advantage of what each component (experimental and numerical) has to offer, is referred to as hybrid simulation. [8]

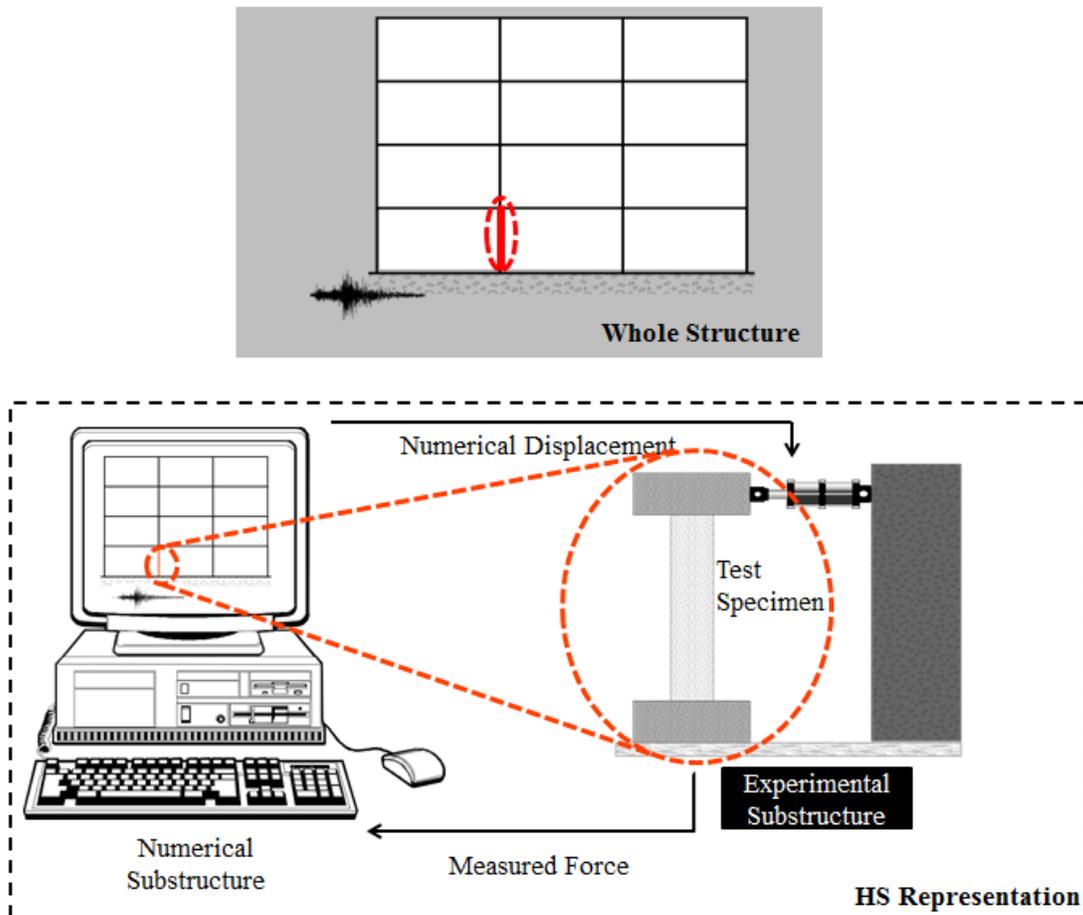


Fig. 1. Whole Structure Model Representation in Hybrid Simulation



Hybrid simulation can enable deeper investigation of complex systems through a broader array of tests than might be performed efficiently at large or full-scale, potentially saving both time and money [8]. Numerous hybrid simulation experiments have been conducted over the past four decades. Within the NEES network at least 29 projects have developed or utilized hybrid simulation methods [13]. Furthermore, recent advances in real-time computing and control methods have accelerated progress toward the development of fast and real-time methods that can better capture rate-dependent effects, coined Real-Time Hybrid Simulation (RTHS). An overview of the contribution of NEES facilities and researches is posted at <https://nees.org/wiki/RTHSwiki>.

Hybrid simulation is intended to obtain useful results while making the most of the experimental resources available. Over the years, the hybrid simulation method has been used for a wide variety of purposes; clearly, there are still many unexplored uses for hybrid simulation. Because hybrid simulation is a relatively new method for much of the research community, and there are such a wide variety of approaches that one can take to perform a hybrid simulation, the steps required to plan and conduct such a test may not be entirely obvious [10]. A researcher should establish clear goals and objectives for the test, and the test should be executed (from planning through to evaluation) with those objectives in mind. Decisions and trade-offs will be made during this process, and assessment measures may be used to evaluate how well the intended goals of the test are achieved. This process is complicated by the fact that in hybrid simulation the ultimate goal is to understand a behavior that is not already well documented or modeled (i.e., there is no reference system or accurate analytical model). Thus, there is no true reference for validation of the hybrid simulation results. Insight must be derived from experiments in which there is a reference or known behavior, and subsequently extrapolated to more complex situations.

To explore effectiveness and accuracy, and thus build broader confidence in the use of this method, there is a need to better understand and address the key features that determine the fidelity and the success in conducting a hybrid simulation. During the meeting, a “successful hybrid simulation” was defined as:



*A system-level simulation that realistically incorporates experimentally-evaluated behavior of elements/members that are difficult to accurately capture in numerical simulation, and where it is not feasible or practical to perform an experiment of the complete system.*

Note that a long-term goal of this effort is to develop what would be termed *acceptance criteria*. Acceptance criteria is a means by which a community can verify that products meet some minimum level of performance and is used in various disciplines. Commonly this term is used for electrical components and systems, and software, or more closely related to earthquake engineering, in shake table testing. To develop such criteria requires developing experience through executing a number of tests to observe the range of performance expected. Acceptance criteria are typically pass or fail, with no intermediate rating. However, with the multitude of possible tests that can be performed, in terms of objectives, configurations, responses, and hardware options would require a great deal of resources, making the development of true acceptance criteria challenging at this time. This report represents a first step in the hybrid simulation community toward that goal.

*The purpose of this document is to provide a summary of the current state of assessment measures that may be implemented to assess the fidelity of a hybrid simulation.* This document is an outcome of the Hybrid Simulation Task Force, following the Hybrid Simulation Primer and Dictionary [8]. This document also describes a variety of assessment methods for hybrid simulation users to evaluate their tests, and is paired with a shared toolkit (HSCAM) of Matlab codes made available to implement these methods at <https://nees.org/resources/hscam/>. Furthermore, we provide a summary of requested future capabilities suggested by the participants of the meeting that would support the broader use of hybrid simulation within the earthquake engineering community.

## **2 Sources of error/uncertainties**

Before we review the methods available in the literature to assess the success of a hybrid simulation, it seems appropriate to discuss possible sources of error. Each hybrid simulation is different, and with experience and planning, these sources of error may be minimized in



conducting hybrid simulations. Thus a protocol is provided in Appendix 2 for conducting a hybrid simulation.

Errors in a hybrid simulation can be attributed to either the numerical components or the experimental components, and even in the steps/connections required to bring the two together. For purposes of this report, errors and uncertainties would include: modeling errors or assumptions made in establishing a numerical model; incompatibilities related to evaluating the equations of motion; and, random and systematic experimental errors related to accurate boundary condition implementation or system delay/lag.

It is important to note that even large errors may be perfectly acceptable. They should be understood and acknowledged, and examined relative to the intended objectives of the particular hybrid simulation.

## **2.1 Assumptions used in numerical models**

A hybrid simulation is influenced by errors originating from how the master model has been substructured or partitioned into the physical and numerical components. Such errors may result from simplifications in modeling and in boundary conditions, be due to limitations in the transfer system used to enforce displacement and ensure compatibility between the numerical and physical components. Errors can also be introduced through the assumptions made in damping of the numerical component as well as any nonlinear material behavior being modeled. If the hybrid simulation in question requires scaling, that may also be a source of certain errors. A brief introduction to these errors is provided here.

## **2.2 Numerical errors/incompatibilities related to the equation of motion**

Hybrid simulation involves the evaluation of the equations of motion of the numerical component. This task is performed through numerical integration techniques solving the equations of motion. As in any such numerical simulation, accumulation of errors may arise due to approximations made in implementing the integration algorithm.



### 2.3 Experimental errors related to random and systematic experimental errors

Unique to a hybrid simulation is the interface between the numerical and physical components of the simulation. This interface, which involves the displacement and force compatibility of the numerical and physical components, is enforced by one or more actuators based on measurements obtained using one or more sensors (e.g. hydraulic actuators and force sensors). Various random and systematic errors may be present. One important source of systematic error involves the accuracy of the actuator control, especially in the case of real-time hybrid simulation. If the actual realized displacement (or force) of the actuator is not exactly what is commanded by the numerical component (and results in a tracking error) then compatibility is not achieved at the boundary. Typically errors between the actual and commanded actuator response are seen in both amplitude and phase, both of which can result in error in the hybrid simulation. Further, when multiple actuators are present, coupling between the actuators may exacerbate the tracking errors. For a hybrid simulation executed at an extended time scale (versus a RTHS), loading and pausing between loading sequences in a hybrid simulation may result in strain-rate effects and force relaxation that introduce errors.

Measurement errors and noise are sources of random errors in hybrid simulation. In most cases in hybrid simulation, the actual restoring force must be physically measured at the interface between the numerical and physical components [8]. The sensor(s) used to measure this force will have limitations and noise due to the sensor technology used.

**Remarks:** Some of the errors mentioned here are strongly affected by the closed loop nature of hybrid simulation and might be amplified and dependent on each other. Furthermore, communication delays may result in additional errors. Users are urged to consider these sources and effects in simulations to minimize them, thus improving their capacity to achieve high fidelity hybrid simulations.

## 3 Assessment Measures

A variety of assessment measures representing best practices from the literature as well as the discussions during the meeting are summarized herein. Some of these focus on the local level compatibility at the interface between the numerical and physical components (including time

domain, frequency domain, and energy based analysis), some focus on the system-level or global responses (partitioning effects, equipment compatibility and global stability are necessary to be considered by means of sensitivity and performance index, time domain, frequency domain and energy balance calculation). The categorization of assessment measures discussed in this report is shown in Fig. 2. Also, note that it is assumed that the response is compared to a reference system, which is not possible in all situations.

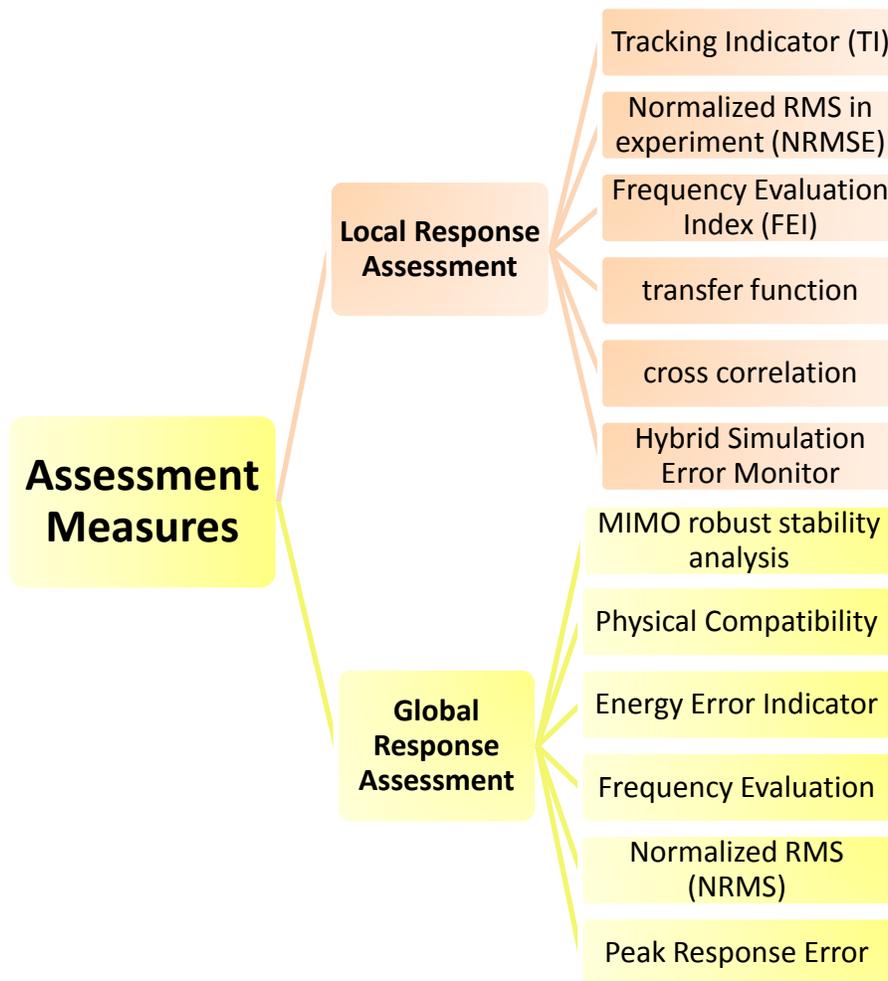


Fig. 2. Assessment Criteria Categorization

### 3.1 Local Response Assessment

Several of the assessment measures focus on the local responses during a hybrid simulation, which consider the physical component and its boundary with the numerical component.

Actuator performance at this interface is of critical importance in hybrid simulation. Here accurate synchronization of the numerical and physical components is a main goal, and best practices are discussed in terms of that class of assessment.

The local performance evaluation is based on boundary condition synchronization between numerical response (target displacement) sent to actuator  $x_c$  and the achieved displacement on the physical component (measured displacement)  $x_m$ . Fig. 3 represents local components in HS, where  $u$  is the actuator command to the actuator after actuator controller/compensator. An example showing the differences between signals  $u$ ,  $x_c$ ,  $x_m$  are illustrated in Fig. 3.

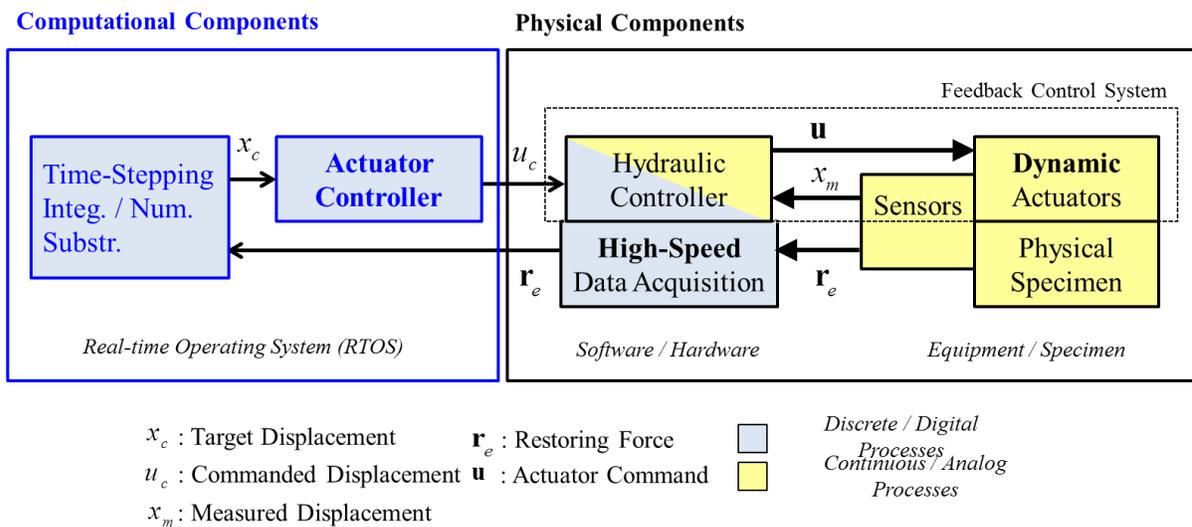


Fig. 3. Local level Components in HS

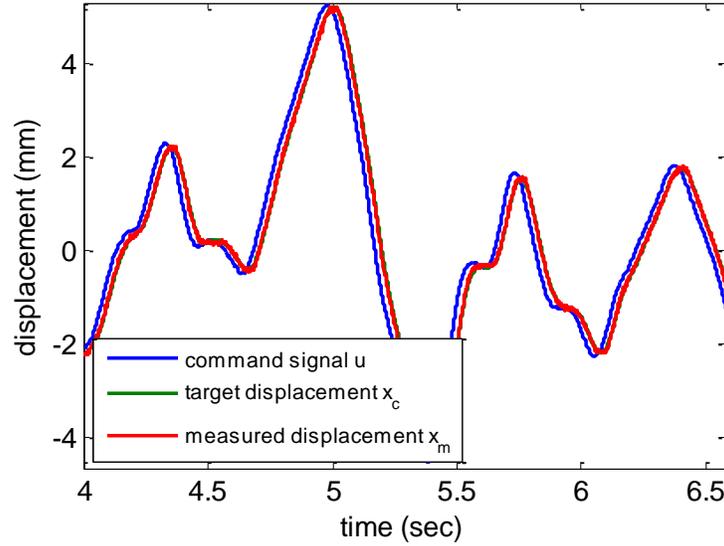


Fig. 4. Illustration of Local Level Displacement Signals in HS

The **tracking indicator** (TI) is used to quantify the difference between the target and measured displacement of the actuator at each time step. The outcome provides a measure of the error at each time step in the hybrid simulation [7]. The nomenclature of each variable is defined in the appendix.

$$TI(i + 1) = \frac{A(i + 1) - TA(i + 1)}{2} \quad (1)$$

$$A(i + 1) = A(i) + \frac{[x_c(i + 1) + x_c(i)][x_m(i + 1) - x_m(i)]}{2} \quad (2)$$

$$TA(i + 1) = TA(i) + \frac{[x_c(i + 1) - x_c(i)][x_m(i + 1) + x_m(i)]}{2} \quad (3)$$

A **normalized root mean square in experiment (NRMSE)** can be used to obtain a single value representing this difference between the measured and target displacements of the actuator. These two measures are time domain measures of the actuator tracking [3, 9].

$$NRMSE \text{ error} = \sqrt{\frac{\sum_{i=1}^N [x_m(i) - x_c(i)]^2}{\sum_{i=1}^N x_c(i)^2}} \quad (4)$$

A frequency domain measure of the actuator tracking is provided by the **frequency evaluation index (FEI)** [4].

$$FEI = \sum_{j=1}^N \left\{ \frac{fft(x_m)_j}{fft(x_c)_j} \cdot \frac{\|fft(x_c)_j\|^l}{\sum_{i=1}^N \|fft(x_c)_i\|^l} \right\} \quad (5)$$

$$f^{eq} = \frac{\sum_{j=1}^N \{\|fft(x_c)_j\|^l \cdot f_j\}}{\sum_{j=1}^N \|fft(x_c)_j\|^l} \quad (6)$$

(7)

$$A_0 = \|FEI\| \quad (8)$$

$$\phi = \arctan[Im(FEI)/Re(FEI)] \quad (9)$$

$$\delta = -\frac{\phi}{2\pi f^{eq}} \quad (10)$$

The **transfer function** can be applied to describe the relationship between the input and output of a linear time invariant system, for instance, a hydraulic actuator. A signal with a wide frequency spectrum (for example, band-limited white noise) can be applied as input ( $u(t)$ ), and output ( $y(t)$ ) is measured correspondingly [3, 9]. The estimated transfer is denoted as:

$$G(s) = Y(s)/U(s) \quad (11)$$

where  $s$  indicates the Laplace variable

$$Y(s) = \mathcal{L}(y(t)), U(s) = \mathcal{L}(u(t)), \quad (12)$$

$\mathcal{L}$  is the Laplace transform operator.

The mathematical form of the magnitude is expressed as

$$Mag = |G(s)| \text{ and } \varphi_G = \arg(G)$$

An ideal transfer system after compensation should aim for  $|G(s)| = 1$  and  $\varphi_G = 0$ , indicating the system output/input magnitude ratio is unity and the lag/delay between input and output is zero.

The **cross correlation** between the target and measured displacement can also provide frequency domain insight into the performance of actuator tracking. Cross-correlation quantifies the degree of similarity between two time series. In hybrid simulation, the cross-correlation between the target and measured displacements provides a reasonable estimate of time delay in the actuator. The mathematical form of the cross-correlation, as well as the estimated time delay from the cross-correlation, are given below.

$$[x_c * x_m](\tau) = \int_{-\infty}^{\infty} x_c(\tau) x_m(t - \tau) d\tau \quad (13)$$

$$\tau_{est} = \arg \max\{[x_c * x_m](\tau)\} \quad (14)$$

Another measure of actuator tracking and any associated error considers energy balance as in the **energy method (EM)** [5, 6]. One of the main goals of a hybrid simulation is to identify the structural properties of the experimental substructure, thus an effort should be made to ensure that these properties are accurately captured in the simulation. Mosqueda et al. proposed the use of energy to quantify the difference between actual experimental behavior (measured forces versus measured displacements) and that observed in explicit numerical simulations (measured forces versus desired displacements).

$$E_E^{err} = E_E^O - E_E = \int F_m^T dx_c - \int F_m^T dx_m \quad (15)$$

where  $E_E$  is the actual energy stored in, or dissipated by the experimental substructures calculated at experimental sampling rate,  $E_E^O$  is the energy dissipation observed by the numerical

analysis subsystem calculated at numerical integration rate, and  $E_E^{err}$  is the experimental energy dissipation error not accounted for in the numerical simulation. The computation of experimental energy error was later modified by Ahmadizadeh and Mosqueda [1] to account for the fact that the final displacement at the end of the integration time step can be modified (e.g., operator splitting method), and this final displacement should be considered as the target displacement in the above formulation.

Calculation of actual experimental energy dissipated at experimental clock rate allows for inclusion of interpolation or re-sampling errors in the resulting energy error, which may be significant in simulations with relatively large integration time steps. This equation mainly captures the difference between the desired and measured displacements (such as those resulting from actuator delay and tracking errors). The energy error can be calculated as an overall error by summing all experimental degrees of freedom in the computations, but it can also be calculated independently for each actuator to monitor its individual performance or order to avoid cancellation of experimental energy errors of opposite algebraic signs. This energy error term can be normalized by input energy to give a non-dimensional error indicator that is only dependent on the experimental errors [5, 6]:

$$HSEM = \frac{E_E^{err}}{E_I + E_E^{max}} \quad (16)$$

where:

$$E_I = \int F_m^T dx \quad (17)$$

$$E_E^{max} = \frac{1}{2} x_0^T K^e x_0 \quad (18)$$

The maximum experimental strain energy ( $E_E^{max}$ ) is used in the denominator to prevent the presence of large values of error in the beginning of each simulation, when the input energy is small.  $K^e$  is the initial stiffness matrix of the test structure, and  $x_0$  is an experimental displacement vector, which can be roughly selected as the yield displacement of the experimental substructure. Mosqueda et al. [6] showed that one can limit the displacement and force errors of



a hybrid simulation by limiting the amount of HSEM. Since the majority of errors in a hybrid simulation are likely from experimental sources, the HSEM is a suitable choice for monitoring simulation quality.

### 3.2 System-level, or Global Response Assessment

Partitioning of the system into numerical and physical components can result in substructuring assumptions that yield errors. If the transfer system is not able to fully capture the boundary conditions at the interface, the sensitivity of the response should be examined to fully understand the effect of locking this degree-of-freedom.

Having a stability and performance indicator for the closed loop system in the presence of transfer system dynamics is critical. A robust stability analysis can provide insight into both the stability and performance. A multi-input multi-output (MIMO) robust stability analysis can be used to provide sufficient conditions for robust stability and robust performance as:

$$\|T_o(s)\Delta(s)\|_\infty < 1 \quad \text{for robust stability} \quad (19)$$

$$\|T_o(s)\Delta(s)\|_\infty \ll 1 \quad \text{for robust performance} \quad (20)$$

where:

$$T_o(s) = [I + P(s)N(s)]^{-1} P(s)N(s) \quad (21)$$

Where  $P(s)$  is a transfer function matrix representation of the physical component,  $N(s)$  is a transfer function matrix representation of the numerical substructure, and

$$\Delta(s) = \hat{A}(s) - I \quad (22)$$

Where  $\hat{A}(s)$  is the transfer function matrix representation of the actuator dynamics with any associated compensation.



Equipment capacity is also important to the tests to be conducted can be accommodated, in capacity, stroke, etc. To ensure capacity in the hybrid test it is important to verify that the equipment enforcing the boundary conditions can meet the demands of the proposed test as given by:

$$F_{act} \geq \max(F_c) \quad (23)$$

$$X_{act} \geq \max(x_c) \quad (24)$$

$$\dot{X}_{act} \geq \max(\dot{x}_c) \quad (25)$$

where  $F_{act}$  is the maximum actuator force,  $X_{act}$  is the peak actuator displacement, and  $\dot{X}_{act}$  is the peak actuator velocity.

The use of numerical integration to solve the equations of motion can result in errors in hybrid simulation. The energy method can be applied at the system level as well to observe the error from an energy perspective. Energy methods are extended to capture both numerical and experimental errors. Filiatrault et al. [2] proposed the use of an energy balance equation to estimate the extent of numerical errors in nonlinear seismic analyses, and showed that it can be a more appropriate measure than comparing peak response parameters, such as displacements and accelerations. In addition, the terms in the energy balance equation can be computed for a hybrid simulation, while the peak response parameters cannot be compared since the “exact” solution is not known. The unbalanced energy is used in this study to develop a normalized error indicator for online assessment of simulation accuracy. In order to include both numerical and experimental errors in this index, the energy balance evaluation procedure is modified, as described next.

The energy balance equation of a simulation can be obtained by integrating the equation of motion over displacement [2]:

$$E_I = E_K + E_D + E_S + E_E^c \quad (26)$$



in which  $E_K$  is the kinetic energy of numerical mass,  $E_D$  is the energy dissipated through viscous damping in numerical substructure,  $E_S$  is the strain energy stored or dissipated in numerical substructure:

$$E_K = \frac{1}{2} \dot{x}_c^T M \dot{x}_c \quad (27)$$

$$E_D = \int \dot{x}_c^T C dx_c \quad (28)$$

$$E_S = \int x_c^T K dx_c \quad (29)$$

and  $E_E^c$  is the energy stored or dissipated in the experimental substructure as observed by the numerical integrator.

Both numerical and experimental errors affect how well the energy balance is maintained. For example, experimental errors make experimental energy  $E_E$  differ from  $E_E^c$  used in numerical analysis to satisfy the equation of motion. On the other hand, numerical truncation errors or relaxed convergence tolerances may result in small differences between left- and right-hand sides of Equation (26). Hence, an overall energy error can be defined as:

$$E^{err} = E_I - (E_K + E_D + E_S + E_E) \quad (30)$$

where  $E_E^c$  is replaced by its actual value  $E_E$ . Within the requirement of engineering precision, and if the convergence tolerance for the integration algorithm is sufficiently small, the energy error obtained from Equation (30) will be very close to  $E_{EA}^{err}$  from Equation (15). That is, it essentially includes the difference between actual experimental and converged energies, when the experiment and the numerical simulation are in phase. Particularly, it cannot capture all of the errors of numerical integration procedure, since most integration methods satisfy the equation of motion and its integral form (Equation (26)).

$$u_n = u_{n-1} + \Delta t v_{n-1} + \left(\frac{1}{2} - \beta\right)(\Delta t)^2 a_{n-1} + \beta(\Delta t)^2 a_n \quad (31)$$

$$v_n = v_{n-1} + (1 - \gamma)\Delta t a_{n-1} + \gamma\Delta t a_n \quad (32)$$

However, satisfying the equation of motion is not sufficient for an accurate and stable simulation; the numerical simulation procedure should also maintain proper kinematic relations between displacement, velocity and acceleration. For example, in the explicit Newmark integration method, the acceleration vector in each step is directly calculated from the equation of motion, but the displacement is not updated according to the acceleration at the current step [ $\beta=0$  in Equation (31-32)]. As a result, equilibrium is satisfied, but the accurate kinematic relations among the states are not fully enforced, leading to conditional stability of this integration method. In order to include the kinematic errors that may occur in the numerical simulation module of hybrid simulation, it is proposed to replace the velocity in Equations (27) and (28) by the first derivative of displacement:

It should be noted that in each integration step, the velocity in Equations (27) and (28) is obtained from Equation (35), while  $\ddot{\mathbf{u}}$  in Equations (15) and (16) is calculated as:

$$E_k = \frac{1}{2} x_c^T M x_c \quad (33)$$

$$E_D = \int \dot{x}_c^T M \dot{x}_c \quad (34)$$

$$\dot{x}_c(i+1) = \frac{x_c(i+1) - x_c(i)}{\Delta t} \quad (35)$$

With this modification, any error in the kinematic relation between displacement and velocity (and hence, between displacement and acceleration) will be reflected as a discrepancy of kinetic and damping energies from those satisfying Equation (26) That is, the energy error given by Equation (30) will also include the effects of differences of velocities and accelerations with time-derivatives of displacements. While this difference may seem insignificant, it was



demonstrated that the velocities given by Equation (32) and Equation (30) can vary substantially in explicit integration methods. Similar to Equation (16), a non-dimensional **energy error indicator (EEI)** can be calculated based on overall unbalanced energy [1]:

$$EEI = \frac{E^{err}}{E_1 + E_E^{max}} \quad (36)$$

A frequency domain approach can be used to examine the global response comparison to some reference. The reference may be an actual structures response, a shake table test, or a pure numerical simulation. The Fourier Spectrum of responses provides this measure. Fourier transform decomposes a time domain sequence into frequency components [10].

$$X_t(k) = \frac{1}{N} \sum_{j=0}^{N-1} x_t(j) e^{-i(2\pi)k \frac{j}{N}} \quad (k=0, \dots, N-1) \quad (37)$$

where  $N$  is the window size chosen in FFT,

$x_t(j)$  is target displacement (global response) in RTHS

$X_t(k)$  is a series of complex numbers

It gives a qualitative assessment of frequency response in RTHS by comparing the amplitude of  $X_{t,RTHS}$  to the amplitude of  $X_{t,sim}$ .

In the time domain, the NRMS error can be used. [3, 9]

$$NRMS \text{ error} = \sqrt{\frac{\sum_{i=1}^N [x_m(i) - x_r(i)]^2}{\sum_{i=1}^N x_r(i)^2}} \quad (38)$$

where  $x_r$  and  $x_m$  are the reference and measured responses at the  $i$ -th time index, respectively.

Similarly in the time domain a peak error in specified responses might be off interest. In order to quantify the error, one can track the observed responses comparing with the predicted numerical solutions or known analytical solutions. The error for peak responses is defined as [3]:



$$e = \left| \frac{x_r^{max} - x_m^{max}}{x_m^{max}} \right| \quad (39)$$

## 4 Example Analysis

Data from two publically available hybrid simulation experiments were used to demonstrate the assessment measures. These tests were conducted at various Network for Earthquake Engineering Simulation (NEES) equipment sites, and these data used in this report are available on the NEEShub.

### 4.1 Performance-Based Design and Real-Time Large-Scale Testing to Enable Implementation of Advanced Damping Systems (Lehigh University)

Advanced structural damping systems such as magneto-rheological (MR) dampers have great potential to accelerate our ability to achieve performance-based structural design (PBD) directed towards seismic resilience. However, developing methods for RTHS of these systems is essential to enable the validation of these approaches. In this NEESR project, large-scale structural models are tested at the Lehigh RTMD NEES Equipment Site using RTHS techniques with specific goals in mind: (a) develop performance-based design methodologies for advanced damping systems; (b) develop and validate real-time large-scale testing techniques; (c) develop higher fidelity models for devices and improved control algorithms for model-based simulation study.

The accomplishments of this research requires systematic testing to characterize magneto-rheological (MR) dampers, adaptation of RTHS for use with the MR dampers and validation testing of semi-active controllers using MR dampers. Furthermore, a selection of prototype structure using Performance-based Design methodologies, design and fabrication of the structural members, development of a linear FEM of the structural model for use in simulations and control design, as well as for use in RTHS are other important outcomes of the project. The experimental setup of this project includes a 30 feet tall / 15 feet wide large-scale damper-braced frame (DBF) and MR dampers as the physical substructure, whereas the gravity system and moment resisting frame (MRF) are modeled as analytical substructures. These components are



connected to each other via a transfer system consisting of the servo-hydraulic motion control systems, hydraulic actuators and real-time target.

The data used in this experiment are from the NEES Project Warehouse, Project 648, Hybrid simulation 3. The path to the data is <https://nees.org/warehouse/hybrid/3807/project/648>) and DOI: 10.4231/D3QJ77Z3Q. More information about this experiment can be found on the project page: <https://nees.org/warehouse/project/648>. Example assessment measure result can be generated using the HSCAM toolkit. [14]

#### **4.2 Semiactive Control of Nonlinear Structures (University of Colorado at Boulder)**

Magneto-rheological (MR) fluid dampers have been identified as a particularly promising type of semiactive control device for hazard mitigation in civil engineering structures. Large-scale experimental testing is important to verify the performance of MR fluid dampers for seismic protection of civil structures. Real-time hybrid testing, where only the critical components of the system are physically tested while the rest of the structure is simulated, can provide a cost-effective means for large-scale testing of semiactive controlled structures.

The Fast Hybrid Test (FHT) facility at the University of Colorado at Boulder (CU) has real-time hybrid simulation capabilities used to experimentally verify large-scale MR fluid dampers. The simulated component in this experiment is a 3 story building structure subjected to suites of ground motions with nonlinear material behavior in the beam elements. The physical component of the experiment is a highly nonlinear and rate-dependant large-scale semiactive MR fluid damper placed between the stories of the simulated building.

The data used in this experiment is from the NEES Project Warehouse, Project 21, Experiment 2. The data is publically available at <https://nees.org/warehouse/experiment/154/project/21>. More information about this experiment can be found on the project page: <https://nees.org/warehouse/project/21>. Example assessment measure result can be generated using HSCAM toolkit [14].



## **5 Recommendations and Request to the Community**

During the course of the conversations at the working meeting, the need for developing a database to collect the quantitative values of the assessment measures included herein (and others proposed) became apparent. Furthermore, a question was posed to the group asking how the cyberinfrastructure can better support the user community in hybrid simulation.

### **Database of Hybrid Simulation Assessment Measures**

To develop acceptance criteria our community will need to gain considerable experience and judgment in the expected values and acceptable ranges of these assessment measures, and how that related to the goals of an experiment. Before clear guidelines could be established regarding the quantitative values of these metrics, a collection of the outcomes from various experiments would contribute to that important step. For instance, the use of a DATASTORE to establish a database to accumulate the various assessment measures from various experiments would be quite useful. As each researcher plans, executes and analyzes a hybrid simulation, these metrics would be documented for various experiments. It was also proposed that MATLAB functions might be available to automatically compute these quantities. The outcomes of those automated tools might be dropped into EVERNOTE (diary style by the graduate student) or some similar tool for easy documentation and entering into the database. These would be useful to establish commonalities between various types of experiments, and to identify a useful range of values for "acceptance" of a HS result.

### **How can NEEShub better meet the needs of the HS community?**

This question was posed to the group and many ideas were generated. These include:

- tool to visualize and insert model, sensors and enter connectivity between physical and numerical structures (this might also support/simplify data upload)
- ability to use AutoCAD files, read directly to ingest metadata from drawings
- import sensor locations into indeed, and link to data files
- online calculation of the metrics using imported information
- online plotting integrated into the cyber-infrastructure



- online comparison to reference model, numerically and visually
- integrated (physical and numerical) animations of the whole structure
- matlab functions available on the NEEShub for assessment calculations

**Integrated Visualization During Testing and Data Upload** would support researchers by increasing impact and saving time and effort. Viewing hybrid simulation results currently requires significant work, and many researchers are duplicating efforts. Researchers need to be able to show the results/performance of a hybrid simulation. Right now, RDV can be used to view the sensor data and numerical results, independently, *during* an experiment. But an integrated, visual view of the data and simulation outputs is not yet available. Experimental results including videos could also be integrated into such a view, with proper synchronization and time scaling.

**Analysis Tools** are needed to better support the users while conducting hybrid simulation, and also for conveying the outcomes of experiments. It would be helpful for NEEScomm to develop tools that support researchers to readily integrate, analyze and plot their data. The user might select columns of data, and automatically generate a tracking plot, a stability plot, etc to view the hybrid simulation results. Comparisons to a reference simulation would be supported, as well as assessment of controller performance in the case of RTHS. Online calculation, and even automated reporting, of the performance metrics would be advantageous. This type of automated analysis and posting of the results would enable more rapid adoption and broad use of hybrid simulation methods.

**Community Bootcamps** should be hosted by the NEES cyberinfrastructure organization on how to develop better visual tools; videos, synchronized with the data plots), are needed by this group so that graduate students can very easily make a video of their experiments to show to funding agencies, other researchers, practitioners, and the public.

These recommendations and user requirements were provided by the participants of the workshop.



## 6 Summary

*The purpose of this document is to provide a summary of the current state of assessment measures that may be implemented to evaluate the fidelity of a hybrid simulation.* To explore effectiveness and accuracy, and thus build broader confidence in the use of this method, there is a need to better understand and address the key features that determine the success in conducting a hybrid simulation. A summary of the discussions during the meeting of the Task Force on Hybrid Simulation is provided, including details of the various proposed assessment measures currently in use by the community. Example data has been identified, and used to demonstrate the use of these assessment measures with real hybrid simulation data. Appendix 2 provides a protocol for planning and conducting a hybrid simulation to maximize success and fidelity in the experiments. A HSCAM (current assessment measures) toolkit for computing the values of these assessment measures in matlab is posted for public use at: <https://nees.org/resources/hscam>.

Note that a long-term goal of this effort is to develop what would be termed *acceptance criteria*. This report is the first step of the hybrid simulation community toward that goal. We also summarize future capabilities suggested by the participants of the meeting that would support the broader use of hybrid simulation within the earthquake engineering community.

## References

1. Ahmadizadeh, M. and Mosqueda, G. (2009) "Online energy-based error indicator for the assessment of numerical and experimental errors in a hybrid simulation." *Engineering Structures*, 31(9), 1987-1996.
2. Filiatrault A, Leger P, Tinawi R. (1994) "On the computation of seismic energy in inelastic structures". *Engineering Structure*, 16(6):425-436.
3. Gao, X. (2012) "Development of a Robust Framework for Real-Time Hybrid Simulation: From Dynamical System, Motion Control to Experimental Error Verification", *Doctoral Dissertation*, Purdue University, West Lafayette, IN.
4. Guo, T., Chen, C., Xu, W., and Sanchez F., "A frequency response analysis approach for quantitative assessment of actuator tracking for real-time hybrid simulation", *Smart Materials and Structures*, 23(4), 2014, doi:10.1088/0964-1726/23/4/045042
5. Mosqueda, G., Stojadinovic, B., and Mahin, S. A. (2007), "Real-time error monitoring for hybrid simulation. Part I: Methodology and Experimental Verification." *Journal of Structural Engineering*, 133(8), 1100-1108.



6. Mosqueda, G., Stojadinovic, B., and Mahin, S. A. (2007), "Real-time error monitoring for hybrid simulation. Part II: Structural response modification due to errors." *Journal of Structural Engineering*, 133(8), 1109-1117
7. Mercan, O., and Ricles, J. M. (2007) "Stability and accuracy analysis of outer loop dynamics in real-time pseudodynamic testing of SDOF systems." *Earthquake Engineering and Structural Dynamics*, 36(11), 1523-1543
8. Naru Nakata; Shirley Dyke; Jian Zhang; Gilberto Mosqueda; Xiaoyun Shao; Hussam Mahmoud; Monique Hite Head; Michael Bletzinger; Gemez A. Marshall; Gaby Ou; Cheng Song (2014) "Hybrid Simulation Primer and Dictionary," <https://nees.org/resources/7702>.
9. Phillips, B. M. and Spencer, B. F. (2011) "Model-Based Feedforward-Feedback Tracking Control for Real-Time Hybrid Simulation", *Newmark Structural Engineering Laboratory Report Series*, University of Illinois at Urbana-Champaign, Urbana, IL, No. 28.
10. Shirley J. Dyke; Naru Nakata; Khalid Mosalam; Carol Shield; James Ricles; Billie F. Spencer (2013), "Hybrid Simulation Survey Report, July 2013," <https://nees.org/resources/7157>.
11. Terras, Audrey, (2009) "Fourier analysis on finite groups and applications", Cambridge University Press, ISBN 978-0-521-45718-7
12. Hybrid Simulation and Real-time Hybrid Simulation Resources in the NEEShub, <https://nees.org/topics/RTHSwiki>
13. D. Gomez, S.J. Dyke and A. Maghareh. "On the Role of Hybrid and Real-time Hybrid Simulation in Advancing the Practice of Earthquake Engineering," *Proceedings of the 6th World Conference on Structural Control and Monitoring*, Barcelona, SPAIN July 2014.
14. Hybrid Simulation Current Assessment Measures (HSCAM) toolkit, <https://nees.org/resources/hscam>.



## Appendix 1. Nomenclature

$x_c$ : Target displacement, also known as commanded displacement, uncompensated displacement, desired displacement [7]

$x_m$ : Measured displacement

$t$ : Time vector

$f$ : Frequency vector

$l$ : Integer power

$F_m$ : Measured force

$TI$ : Tracking Indicator

$A(i)$ : Enclosed area for the  $i$ -th step (tracking indicator)

$TA(i)$ : Enclosed complementary area for the  $i$ -th step(tracking indicator)

$NRMS\ error$ : Normalized root mean square

$FEI$ : Frequency evaluation index

$f^{eq}$ : Equivalent frequency

$A_0$ : Generalized amplitude

$\Phi$ : Generalized angle phase

$\delta$ : Equivalent delay

$x_c * x_m$ : Convolution of target and measured displacement records

$\tau_{est}$ : Estimated time delay

$e$ : Peak error

$HSEM$ : Hybrid Simulation Error Monitor

$E_E^{err}$ : Experimental energy dissipation error

$E_E$ : Energy dissipated by the experimental substructure

$E_E^O$ : Energy dissipated by the experimental substructure observed by the numerical analysis subsystem

$E_E^C$ : Energy dissipated in the experimental substructure as observed by the numerical integrator

$E_I$ : Input energy

$E_K$ : Kinetic energy



$E_D$ : Energy dissipated through viscous damping in numerical substructure

$E_S$ : Strain energy of the numerical substructure

$E_E^{max}$ : Maximum experimental strain energy

$x_0^T$ : Experimental displacement vector

$K^e$ : Initial stiffness matrix of the test structure

$F_{act}$ : Maximum actuator force

$X_{act}$  Peak actuator displacement

$\dot{X}_{act}$  Peak actuator velocity

$x_r$  Reference system simulation response



## Appendix 2. Hybrid Simulation Protocol

Hybrid simulation will typically require developing a customized combination of software and laboratory hardware that is unique to each series of tests. As such, it is important to conduct step-by-step checks to ensure all components and communications links are working properly prior to running an actual test. In addition, the performance of the equipment should be verified during and after a test to assess the quality of the results. The suggested testing protocol for hybrid simulation (HS) involves steps prior to (pretest), during and after the actual test.

### A2.1 Pretest stage

Prior to performing an actual HS, some pre-test tasks are suggested to ensure a successful test:

- **Preliminary Simulation of Prototype:** In this step a whole structure simulation should be conducted numerically to generate a baseline response for comparison and to identify, even at a preliminary level, the anticipated requirements of the system test including actuators.
- **Partitioning:** Because HS involves numerical and physical components, a critical decision is identifying the numerical and physical components of the partitioned system. Especially if the structural frame is partitioned, this will certainly affect the complexity and demands required of the transfer system enforcing the boundary conditions between the substructures, and will also have an impact on the accuracy and stability of the HS. A simulation of the partitioned model can be conducted with a model of the experiment capturing the actual boundary conditions imposed by the actuators then checked against the full prototype simulation to quantify errors in substructuring.
- **Integration scheme:** The numerical integration scheme is an important component to a HS test, and is needed to evaluate the dynamic response of the numerical component of the test. It is important at this point to evaluate the accuracy of the integration scheme and integration time step chosen for the HS. Given that fully implicit integration algorithms with convergence criteria are not typically applied in a hybrid



simulation, simple checks can include monitoring the unbalanced forces during a simulation.

- **Equipment compatibility and capacity:** The partitioning strategy will determine the boundary conditions between the numerical and physical components and the resulting configuration of the experimental setup. The demands of the equipment applying the displacements and forces must be evaluated to ensure compatibility and equilibrium at the boundaries. The number of degrees-of-freedom, the capacity, the speed and stroke of the actuation system as determined from numerical simulations and the range, sensitivity and noise in the measurements must be all verified. For RTHS, it is also important to check the velocity requirements of the test compared to the capacity of the actuators.
- **Stability/Performance of Actuators:** Stability and performance of the actuators are critical aspects of a HS test. Their performance should be evaluated in the pretest stage as well as an indication of the accuracy, or performance, during the test. For RTHS, it is important to minimize lags/delay while ensuring the stability of the system. Lags/delay will also affect the results of slow tests, though it is often not as critical.
- **Open Loop Tests – No Specimen:** Open loop tests can provide valuable information regarding the basic operation and experimental setup, including the performance of the actuation system, sensor wiring and operation. These tests are conducted without a specimen to verify control commands to the test setup from the hybrid simulation. Without the specimen, the test setup can return feedback displacements but there are no feedback forces. To simulate a hybrid test, the feedback forces can be generated from a numerical model of the test setup. The actual lags/delays of the actuator and their overall performance as observed by the HS algorithm can be estimated from the feedback displacements. Note that this does not apply to actuators under force control.
- **Open loop tests – With Specimen:** Repeating the same test with an actual specimen at low input levels will confirm feedback force measurements are as expected. In this case, the experiment is fully set up, but the measured restoring forces are not used by



the HS, rather these are again generated by a numerical model so that the actual displacement is the same as numerical simulations. The reason for doing this is so that the displacement demands are the same as in the numerical model. This will prevent any instability if the feedback forces are not being captured correctly by the HS algorithms. The following can be examined in the open loop tests to provide overall insight into the ultimate performance of the HS test.

- Actuation system characterization: The actuation system can be a complex nonlinear system dependent on the bandwidth and amplitude of the commanded displacement and the reaction force of the attached specimen. Characterizing the actuation system during the open loop test can provide realistic conditions. System identification (SI) of the actuation system can be conducted to identify the magnitude and phase difference in the input-output relationship of the commanded displacement to measured (actual measured) displacement. SI should be conducted over the bandwidth of interest. The apparent time delay (lag) from actuator dynamics can be determined from the phase determined in SI or by close examination of the displacement commanded and measured time histories.
- Controller design/test/validation: During this phase the controller for the actuation system should be designed (if needed) based on the identified model of the actuator system. The controller can then be tested and verified through open loop tests.
- Actuator performance check: The actuator system, with any additional controller, should be verified to meet the demands of the hybrid simulation, including the anticipated amplitude (stroke), the frequency (bandwidth), and the specimen interaction.
- At this point, the different error measures discussed can be applied to assess the expected errors during actual hybrid simulations.



## **A2.2 HS stage**

The following sequence of tests is suggested to ensure a successful test:

- **Low-level excitation hybrid simulation:** Initially a hybrid simulation may be executed using low levels and demands well below the yield point of the structure. These low levels can be accomplished using a small (scaled) amplitude earthquake, a small impulse as the excitation, or a low-level band-limited white noise excitation. During the low level tests, the performance of the actuation system (controller) can be further verified. This assumes that the amplitude dependence of the nonlinearities of the actuation system is not sufficient. The controller performance can again be checked. The stability and performance of the closed loop hybrid simulation can also be examined.
- **Execute hybrid simulation:** During the actual hybrid simulation various checks can be conducted including online evaluation of data, TI (tracking indicator), and a comparison to the pretest simulation. In particular, for the low-level linear tests can be compared against the numerical simulations.

## **A2.3 Processing stage**

After the hybrid simulation the results can be examined by considering the various error measures to quantify the experimental and numerical errors. Experimental errors can be assessed based on the measures discussed in this paper while numerical errors can be assessed by examining equilibrium errors in the algorithm such as unbalanced forces.