

# Evaluation of 9-Story Benchmark Building Shear Model

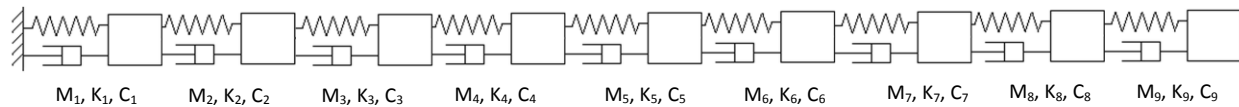


Figure 1. 9 DOF Shear Building Model

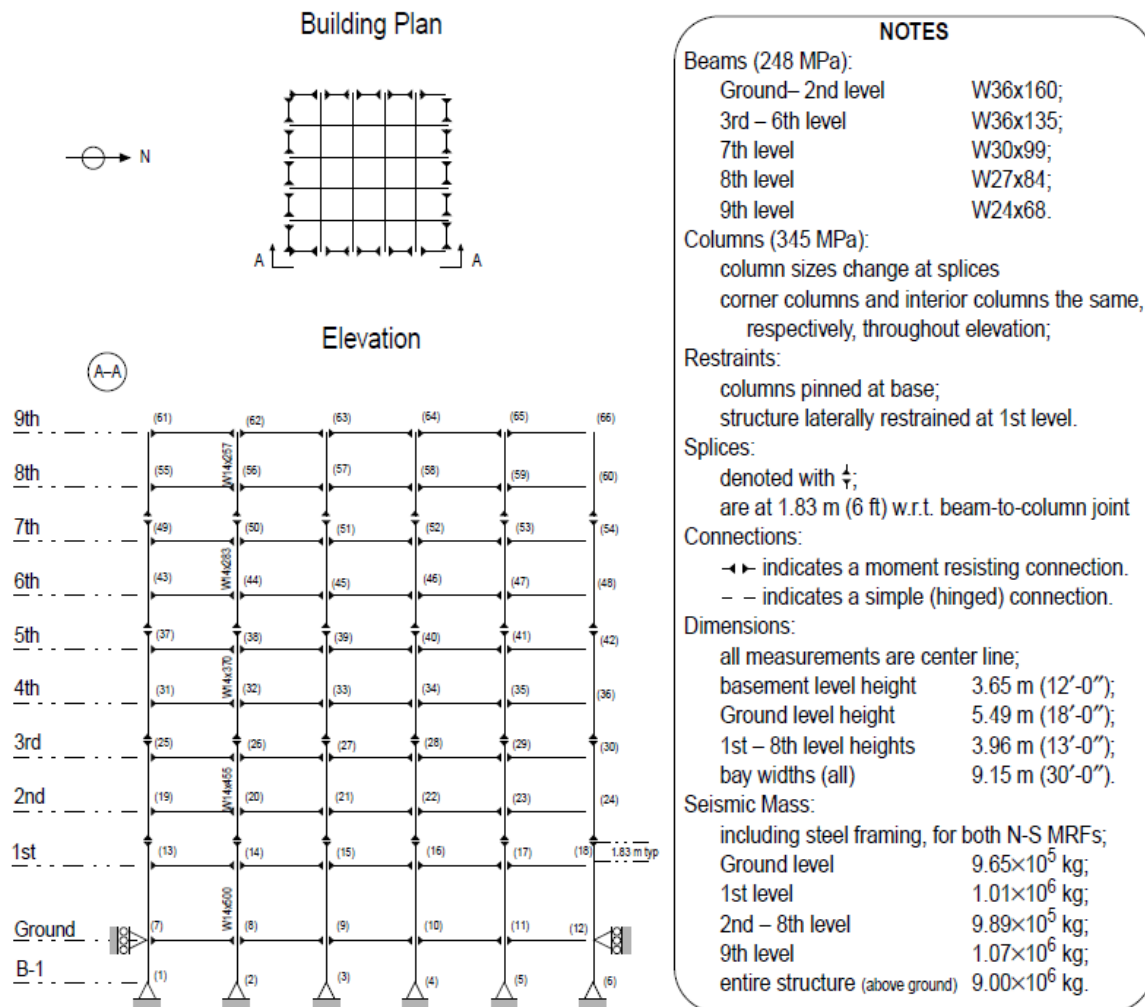


Figure 2. 9-Story Benchmark Building<sup>1</sup>

<sup>1</sup> Y. Ohtori et al., *Benchmark Control Problems for Seismically Excited Nonlinear Buildings*, J. Eng. Mech. 130, 366 (2004)

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## Introducing Mass and Stiffness Matrices

Mass elements are introduced based on the 9-story benchmark FEM model

```
clear all; close all; clc
m1 = 1.01e6; % mass of 1st floor
m2 = 9.89e5; % mass of 2-8th floor
m3 = m2; m4 = m2; m5 = m2; m6 = m2; m7 = m2; m8 = m2;
m9 = 1.07e6; % mass of 9th floor

% Construction of mass matrix
M = [m1 0 0 0 0 0 0 0 0;
      0 m2 0 0 0 0 0 0 0;
      0 0 m3 0 0 0 0 0 0;
      0 0 0 m4 0 0 0 0 0;
      0 0 0 0 m5 0 0 0 0;
      0 0 0 0 0 m6 0 0 0;
      0 0 0 0 0 0 m7 0 0;
      0 0 0 0 0 0 0 m8 0;
      0 0 0 0 0 0 0 0 m9];

% Story Height
L = [5.49, 3.96, 3.96, 3.96, 3.96, 3.96, 3.96, 3.96];

% Cumulative Height
H = cumsum(L);

% Moment of inertia of columns (%90 reduction at splices)
Ic = 1e-3*[3.41726, 0.90*3.1889, 2.9927, 0.90*2.6009, 2.2643,...
           0.9*1.9061, 1.5983, 0.9*1.4998, 1.4152];

% Moment of inertia of beams
Ib = 1e-3*[4.0624, 4.0624, 3.2466, 3.2466, 3.2466,...
           3.2466, 1.6608, 1.1863, 0.7617];

% Modulus of elasticity
E = 200e9;
ro = Ib./(4*Ic);
for i=1:9
    k(i) = (1*(12*(12*ro(i)+1)/(12*ro(i)+4))*E*Ic(i)+...
            4*(12*(12*2*ro(i)+1)/(12*2*ro(i)+4))*E*Ic(i)+3*E*Ic(i))/L(i)^3;
end

% Stiffness matrix
K = [k(1)+k(2) -k(2) 0 0 0 0 0 0 0;
      -k(2) k(2)+k(3) -k(3) 0 0 0 0 0 0;
      0 -k(3) k(3)+k(4) -k(4) 0 0 0 0 0;
      0 0 -k(4) k(4)+k(5) -k(5) 0 0 0 0;
      0 0 0 -k(5) k(5)+k(6) -k(6) 0 0 0;
      0 0 0 0 0 0 0 0 0];
```

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```

0 0 0 0 -k(6) k(6)+k(7) -k(7) 0 0;
0 0 0 0 0 -k(7) k(7)+k(8) -k(8) 0;
0 0 0 0 0 0 -k(8) k(8)+k(9) -k(9);
0 0 0 0 0 0 0 -k(9) k(9) ];

[fi,r] = eig(inv(M)*K);
r = diag(r);
% Natural freq in rad/s
w = sqrt(r);
% Arrange NF in ascending order
[w, index] = sort(w);
% Getting natural freq in Hz
NF = w/(2*pi);
% Rearranging mode shape order in accordance with NF
phi = fi(:,index);

% Normalizing the mode shape to make the Max entry 1 in each column
for i=1:size(fi,2),
    [fac n] = max(abs(phi(:,i)));
    sgn(i,1) = sign(phi(n,i));
    phi(:,i) = sgn(i)*phi(:,i)./fac;
end

% Modal mass
Mn = phi'*M*phi;
% Modal stiffness
Kn = phi'*K*phi;
% Damping ratio
zeta = 0.02;
% Modal damping
Cti=2*Mn.*zeta*...
    [w(1) 0 0 0 0 0 0 0 0;
    0 w(2) 0 0 0 0 0 0 0;
    0 0 w(3) 0 0 0 0 0 0;
    0 0 0 w(4) 0 0 0 0 0;
    0 0 0 0 w(5) 0 0 0 0;
    0 0 0 0 0 w(6) 0 0 0;
    0 0 0 0 0 0 w(7) 0 0;
    0 0 0 0 0 0 0 w(8) 0;
    0 0 0 0 0 0 0 0 w(9)];
% Damping matrix
C = inv(phi')*Cti*inv(phi);
% Modal mass
Cn = phi'*C*phi;
save mode phi

```

## Retrieving the FEM model

```

load Model
[fiF,rF] = eig(inv(MF)*KF);
rF = diag(rF);
% Natural freq in rad/s
wF = sqrt(rF);
% Arranging NF in ascending order
[wF, index] = sort(wF);
% Getting natural freq in Hz
NFF = wF/(2*pi);
NFFEM = NFF(1:9);
%Getting Percentage Error of NF's
NFError = abs((NFFEM-NF)./NF)*100;
% Rearrange mode shape order to correspond to NF
phiF1 = fiF(:,index);

```

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```

for i = 1:9
    phiF2(:,i) = phiF1([23,36,49,62,75,88,101,114,127],i);
end

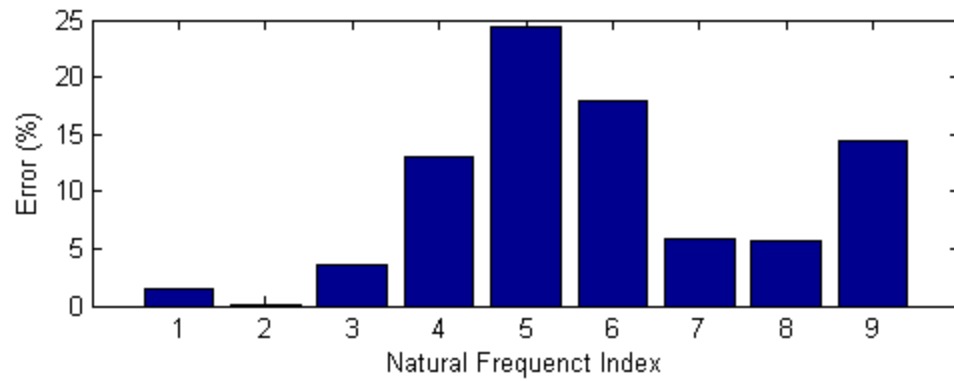
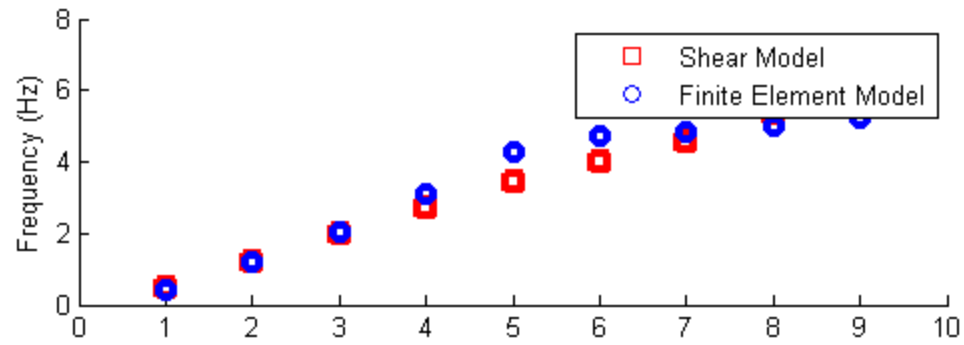
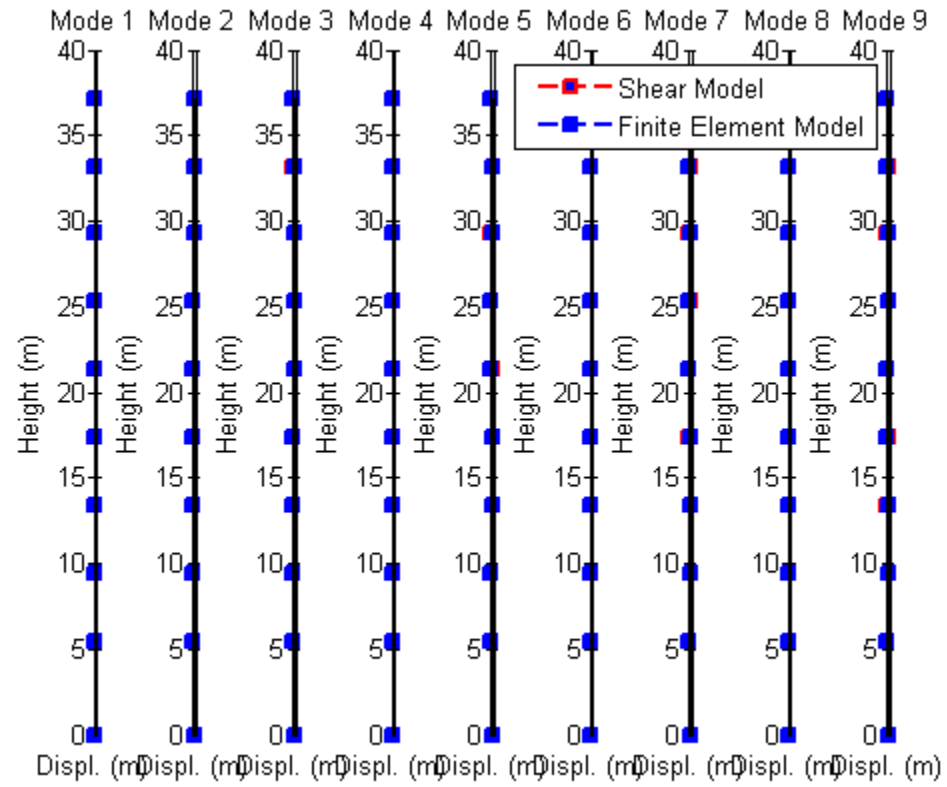
% Normalizing the mode shape to make the Max entry 1 in each column
for i=1:9,
    [facF nF] = max(abs(phiF2(:,i)));
    phiFEM(:,i) = (phiF2(:,i)./facF);
    phiFEM(:,i) = sign(phiFEM(1,i)/phi(1,i))* phiFEM(:,i);
end

figure(1)
h = [0 H];
N = 9;
for i = 1:9;
    subplot(1,9,i); plot([0;phi(:,i)],h,'--rs',...
        [0;phiFEM(:,i)],h,'--bs','LineWidth',2,...
        'MarkerFaceColor','b','MarkerSize',5); hold on;
    plot(zeros(N+1,1),h,'k','LineWidth', 2);
    xlim([-2,2]);
    xlabel('Displ. (m)');
    ylabel('Height (m)');
    title(['Mode ' int2str(i)])
end
legend('Shear Model','Finite Element Model')

figure(2)
subplot(211)
scatter([1:9],NF,'rs','LineWidth',3); hold on;
scatter([1:9],NFFEM,'b','LineWidth',3); xlim([0,10])
ylabel('Frequency (Hz)')
legend('Shear Model','Finite Element Model')
subplot(212)
bar(NFError)
xlabel('Natural Frequenct Index')
ylabel('Error (%)')

```

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## Ground Excitation [Elcentro Intensity = 0.5]

```
fs = 100; Tmax = 60;
load elcentro
load resp_elcentro
t = e(:,1);
% Intensity
int = 0.5;
%Ground Acceleration
zg = int*e(:,2);
np = length(ye(:,1));

% Creating state space form (earthquake form)
AA = [zeros(N) eye(N);-inv(M)*K -inv(M)*C];
% Earthquake input
BB = [zeros(N,1);-ones(N,1)];
CC = [eye(N) zeros(N); zeros(N) eye(N);-inv(M)*K -inv(M)*C];
% Earthquake input
DD = zeros(3*N,1);
sim('Model');

figure(3)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i),'r',t_out,ye(:,3*i),'--b');
    ylim([-0.4,0.4])
    xlabel('Time (sec.)');
    ylabel('displ. (m)')
    title(['Elcentro// floor' int2str(i) ' //Max relative disp. ='...
        num2str(max(yout(:,i))) 'm']);
end
legend('Shear Model','Finite Element Model')

figure(4)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+3),'r',t_out,ye(:,3*(i+3)),'--b');
    ylim([-0.4,0.4])
    xlabel('Time (sec.)');
    ylabel('displ. (m)')
    title(['Elcentro// floor' int2str(i+3) ' //Max relative disp. ='...
        num2str(max(yout(:,i+3))) 'm']);
end
legend('Shear Model','Finite Element Model')

figure(5)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+6),'r',t_out,ye(:,3*(i+6)),'--b');
    ylim([-0.4,0.4])
    xlabel('Time (sec.)');
    ylabel('displ. (m)')
    title(['Elcentro// floor' int2str(i+6) ' //Max relative disp. ='...
        num2str(max(yout(:,i+6))) 'm']);
end
legend('Shear Model','Finite Element Model')

figure(6)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+9),'r',t_out,ye(:,3*i-1),'--b');
    ylim([-0.6,0.6])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)')
    title(['Elcentro// floor' int2str(i) ' //Max relative veloc. ='...
        num2str(max(yout(:,i+9))) 'm/s']);
end
legend('Shear Model','Finite Element Model')
```

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```

        num2str(max(yout(:,i+9))) 'm/s']));
end
legend('Shear Model','Finite Element Model')

figure(7)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+12),'r',t_out,ye(:,3*(i+3)-1),'--b');
    ylim([-0.6,0.6])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)')
    title(['Elcentro// floor' int2str(i+3) ' //Max relative veloc. ='...
        num2str(max(yout(:,i+12))) 'm/s']));
end
legend('Shear Model','Finite Element Model')

figure(8)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+15),'r',t_out,ye(:,3*(i+6)-1),'--b');
    ylim([-0.6,0.6])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)')
    title(['Elcentro// floor' int2str(i+6) ' //Max relative veloc. ='...
        num2str(max(yout(:,i+15))) 'm/s']));
end
legend('Shear Model','Finite Element Model')

figure(9)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+18),'r',t_out,ye(:,3*i-2),'--b');
    ylim([-4,4])
    xlabel('Time (sec.)');
    ylabel('abs. acc. (m/s2)')
    title(['Elcentro// floor' int2str(i) ' //Max abs acc. ='...
        num2str(max(yout(:,i+18))) 'm/s2']));
end
legend('Shear Model','Finite Element Model')

figure(10)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+21),'r',t_out,ye(:,3*(i+3)-2),'--b');
    ylim([-4,4])
    xlabel('Time (sec.)');
    ylabel('abs. acc. (m/s2)')
    title(['Elcentro// floor' int2str(i+3) ' //Max abs. acc. ='...
        num2str(max(yout(:,i+21))) 'm/s2']));
end
legend('Shear Model','Finite Element Model')

figure(11)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+24),'r',t_out,ye(:,3*(i+6)-2),'--b');
    ylim([-4,4])
    xlabel('Time (sec.)');
    ylabel('abs. acc. (m/s2)')
    title(['Elcentro// floor' int2str(i+6) ' //Max abs. acc. ='...
        num2str(max(yout(:,i+24))) 'm/s2']));
end
legend('Shear Model','Finite Element Model')

for i = 1:9
    rmsEl(i,1) = (1/np)*sqrt(sum((yout(:,i))-ye(:,3*i)).^2)); %RMS Error Elcentro
    rmsEl(i,2) = (1/np)*sqrt(sum((yout(:,i+9))-ye(:,3*i-1)).^2)); %RMS Error Elcent
    rmsEl(i,3) = (1/np)*sqrt(sum((yout(:,i+18))-ye(:,3*i-2)).^2)); %RMS Error Elcen

```

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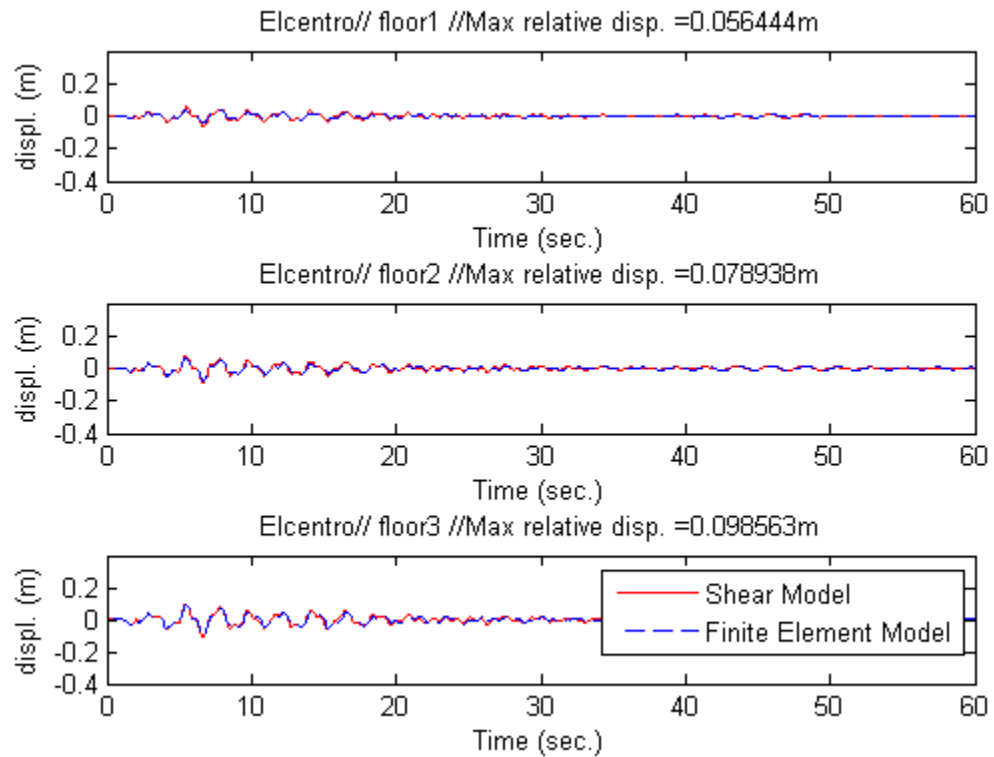
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```

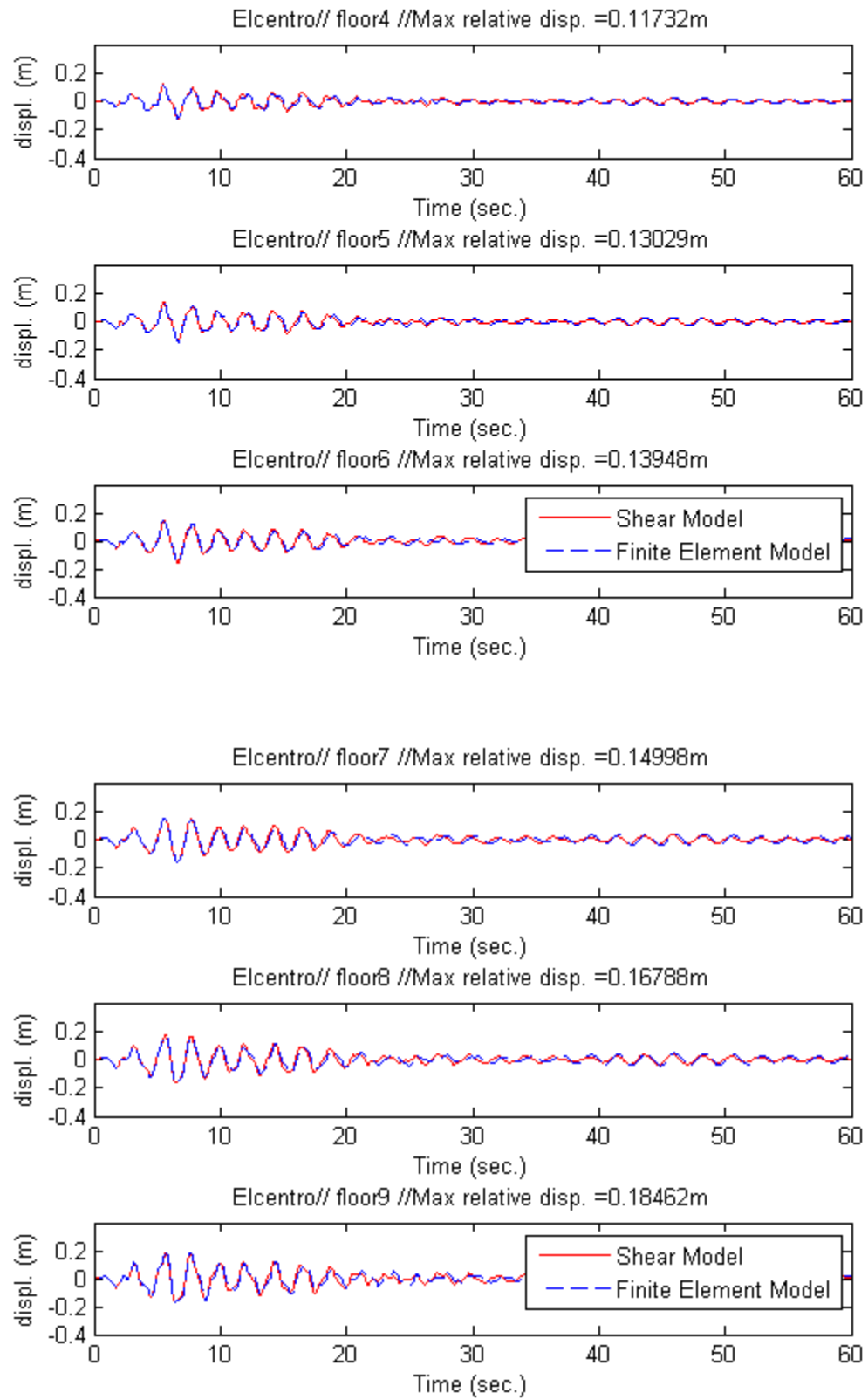
        PRe(i,2) = max(yout(:,i)); %Peak Response Shear Model
        PRe(i,1) = max(ye(:,3*i)); %Peak Response FEM Model
    end
    PRer = abs(PRe(:,1)-PRe(:,2))./(PRe(:,2))*100; %Peak Response Error
    figure(12)
    subplot(311)
    bar(rmsEl,'group');
    legend('Rel. Disp.','Rel. Vel.','Abs. Acc.')
    ylabel('RMS Error (m)')
    title('RMS Error 50% intensity Elcentro')
    subplot(312)
    bar(PRe,'group');
    ylabel('Peak Response (m)')
    title('Peak Response')
    legend('Finite Element Model','Shear Model')
    subplot(313)
    bar(PRer); xlabel('Floor No. ');
    ylabel('Peak Response Error(%)')
    title('Peak Response Error')

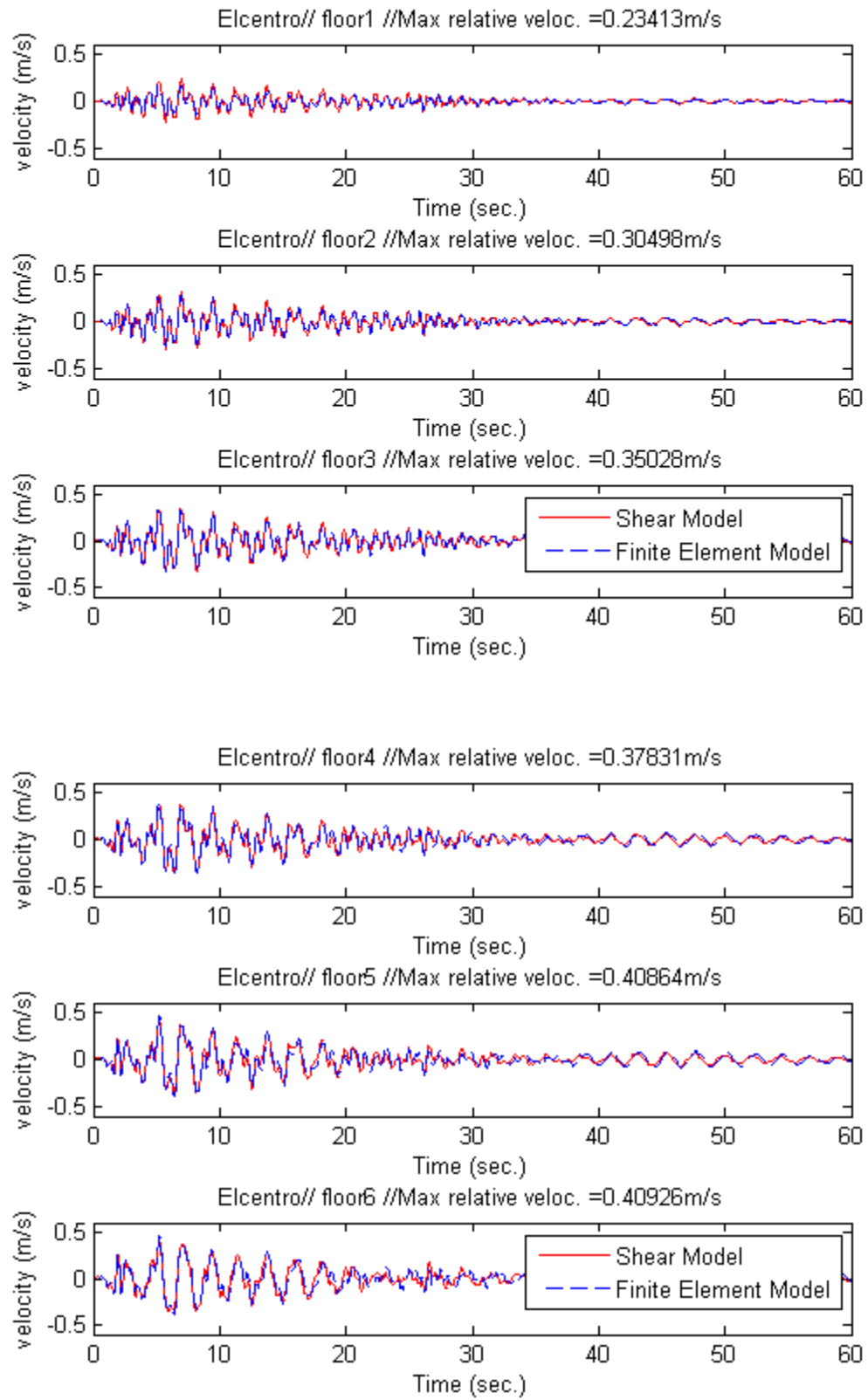
```

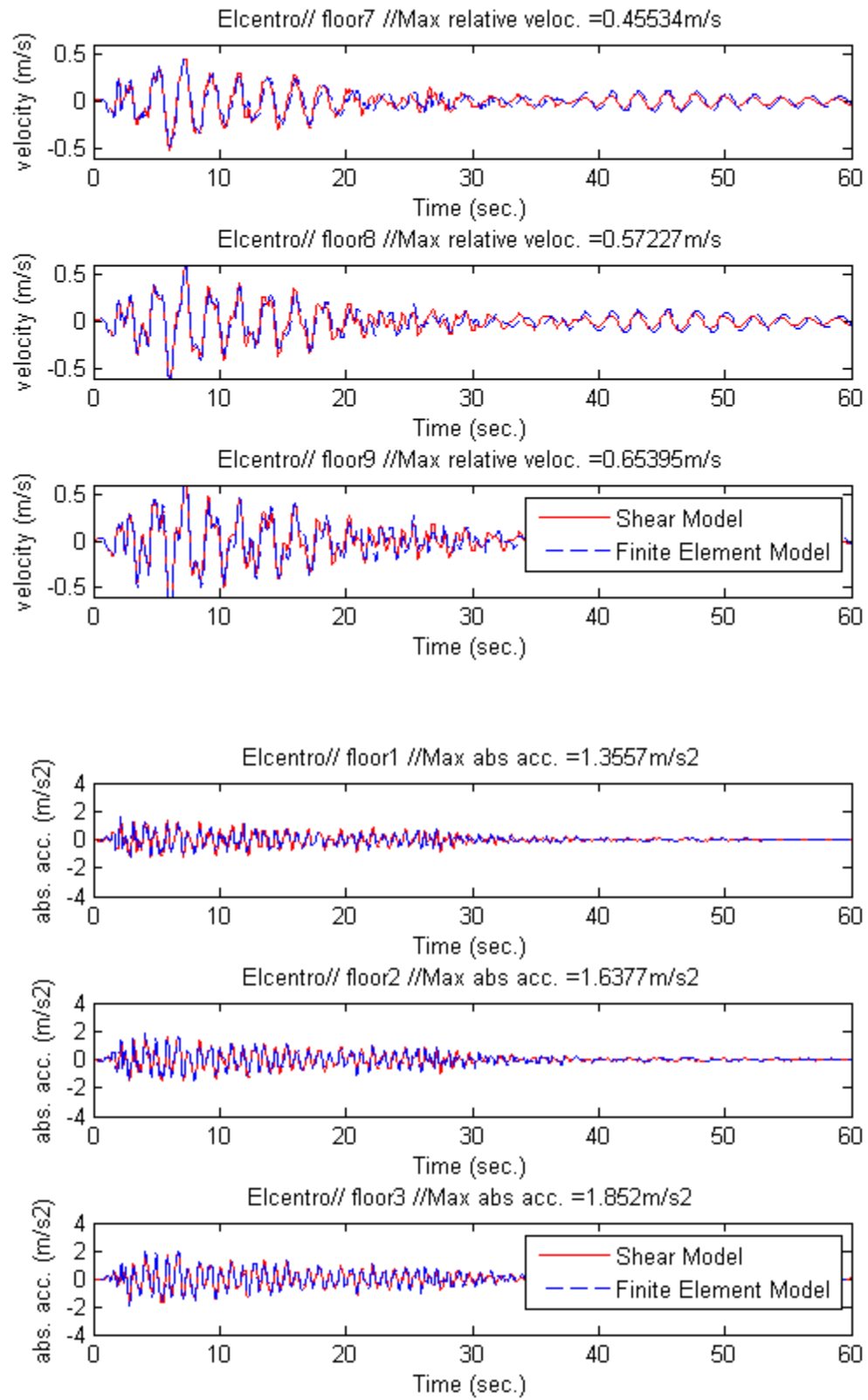
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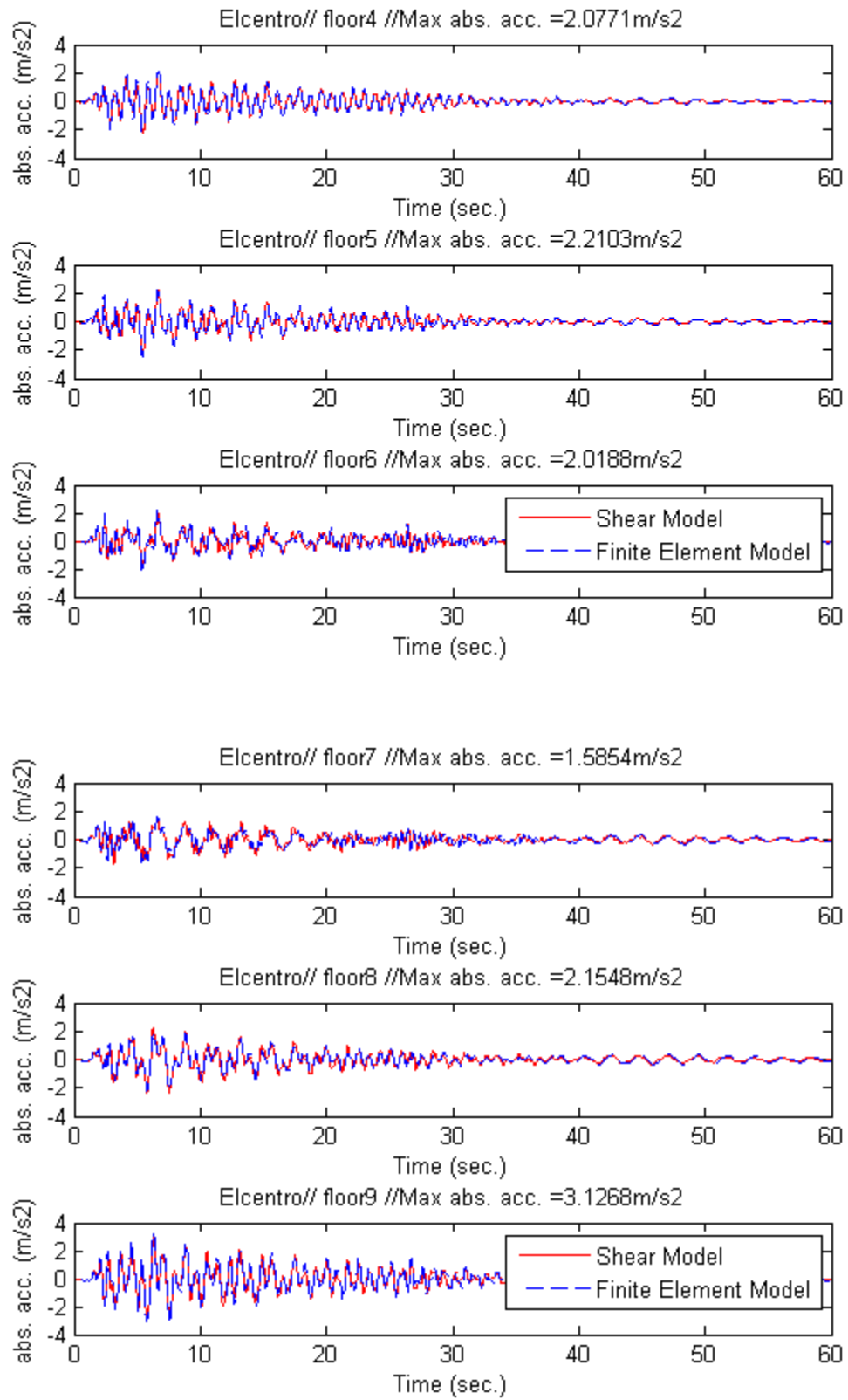


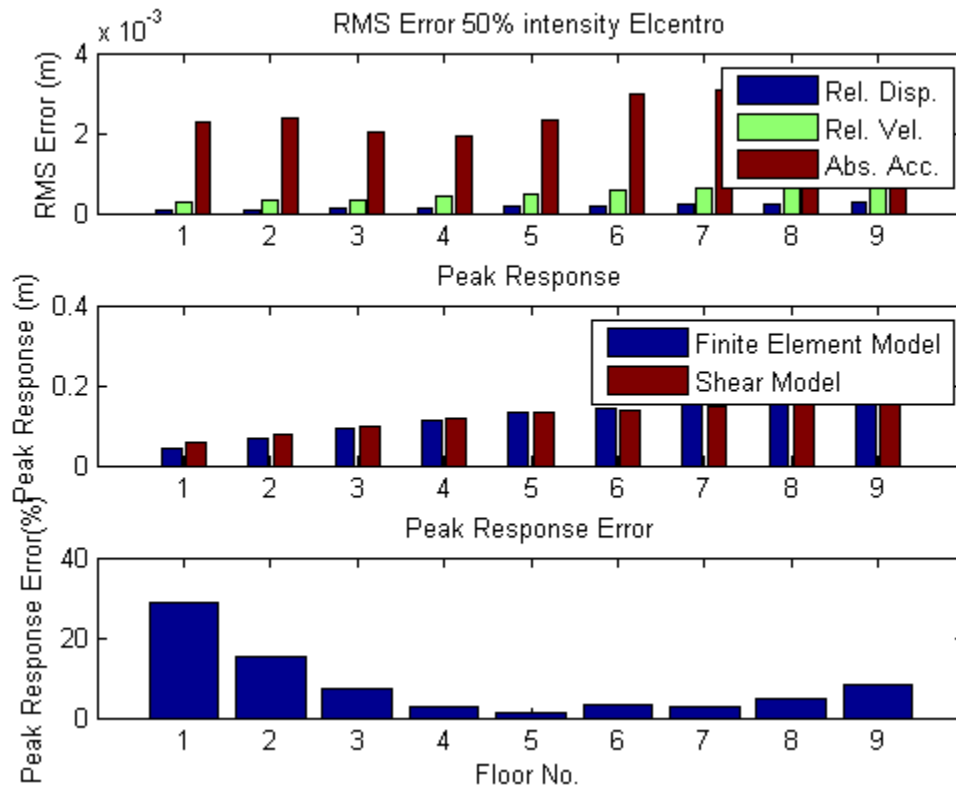












## Ground Excitation [Hachinohe Intensity = 0.5]

```

fs = 100; Tmax = 60;
load hachinohe
load resp_hachinohe
t = h(:,1);
% Intensity
int = 0.5;
%Ground Acceleration
zg = int*h(:,2);
% Creating state space form (earthquake form)
AA = [zeros(N) eye(N); -inv(M)*K -inv(M)*C];
% Earthquake input
BB = [zeros(N,1); -ones(N,1)];
CC = [eye(N) zeros(N); zeros(N) eye(N); -inv(M)*K -inv(M)*C];
% Earthquake input
DD = zeros(3*N,1);
sim('Model');

figure(13)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i),'r',t_out,ye(:,3*i),'--b');
    xlim([0,Tmax])
    ylim([-0.3,0.3])
    xlabel('Time (sec.)');
    ylabel('displ. (m)')
    title(['Hachinohe// floor' int2str(i) ' //Max relative disp. = '...
        num2str(max(yout(:,i))) 'm']);
end
legend('Shear Model','Finite Element Model')

```

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```

figure(14)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+3),'r',t_out,ye(:,3*(i+3)),'--b');
    xlim([0,Tmax])
    ylim([-0.3,0.3])
    xlabel('Time (sec.)');
    ylabel('displ. (m)');
    title(['Hachinohe// floor' int2str(i+3) ' //Max relative disp. ='...
        num2str(max(yout(:,i+3))) 'm']);
end
legend('Shear Model','Finite Element Model')

figure(15)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+6),'r',t_out,ye(:,3*(i+6)),'--b');
    xlim([0,Tmax])
    ylim([-0.3,0.3])
    xlabel('Time (sec.)');
    ylabel('displ. (m)');
    title(['Hachinohe// floor' int2str(i+6) ' //Max relative disp. ='...
        num2str(max(yout(:,i+6))) 'm']);
end
legend('Shear Model','Finite Element Model')

figure(16)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+9),'r',t_out,ye(:,3*i-1),'--b');
    xlim([0,Tmax])
    ylim([-1,1])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)');
    title(['Hachinohe// floor' int2str(i) ' //Max relative veloc. ='...
        num2str(max(yout(:,i+9))) 'm/s']);
end
legend('Shear Model','Finite Element Model')

figure(17)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+12),'r',t_out,ye(:,3*(i+3)-1),'--b');
    xlim([0,Tmax])
    ylim([-1,1])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)');
    title(['Hachinohe// floor' int2str(i+3) ' //Max relative veloc. ='...
        num2str(max(yout(:,i+12))) 'm/s']);
end
legend('Shear Model','Finite Element Model')

figure(18)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+15),'r',t_out,ye(:,3*(i+6)-1),'--b');
    xlim([0,Tmax])
    ylim([-1,1])
    xlabel('Time (sec.)');
    ylabel('velocity (m/s)');
    title(['Hachinohe// floor' int2str(i+6) ' //Max relative veloc.. ='...
        num2str(max(yout(:,i+15))) 'm/s']);
end
legend('Shear Model','Finite Element Model')

figure(19)
for i = 1:3
    subplot(3,1,i); plot(time,yout(:,i+18),'r',t_out,ye(:,3*i-2),'--b');

```

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```

        xlim([0,Tmax])
        ylim([-3,3])
        xlabel('Time (sec.)');
        ylabel('abs. acc. (m/s2)')
        title(['Hachinohe// floor' int2str(i) ' //Max abs. acc. ='...
              num2str(max(yout(:,i+18))) 'm/s2']);
    end
    legend('Shear Model','Finite Element Model')

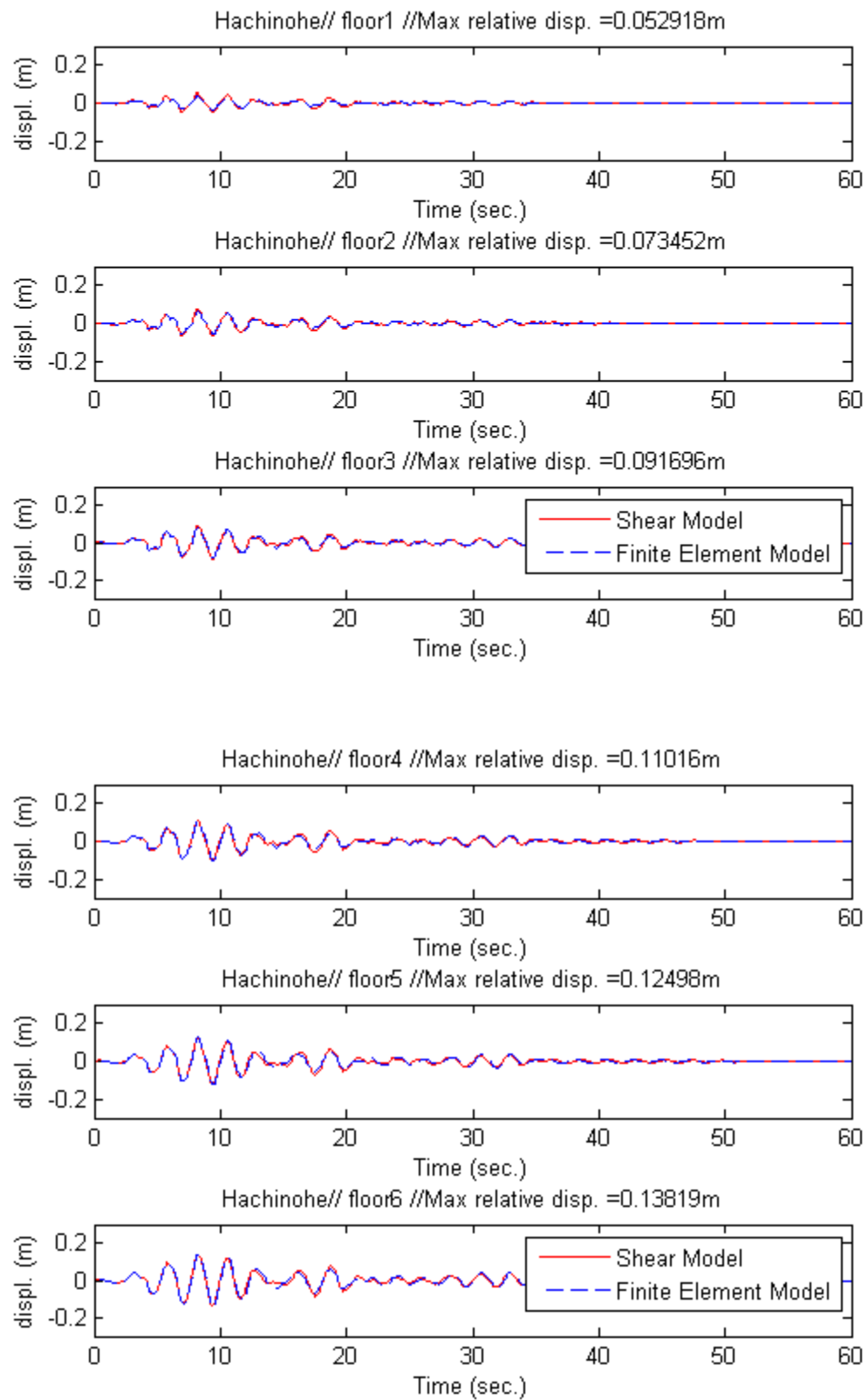
    figure(20)
    for i = 1:3
        subplot(3,1,i); plot(time,yout(:,i+21),'r',t_out,ye(:,3*(i+3)-2),'--b');
        xlim([0,Tmax])
        ylim([-3,3])
        xlabel('Time (sec.)');
        ylabel('abs. acc. (m/s2)')
        title(['Hachinohe// floor' int2str(i+3) ' //Max abs. acc. ='...
              num2str(max(yout(:,i+21))) 'm/s2']);
    end
    legend('Shear Model','Finite Element Model')

    figure(21)
    for i = 1:3
        subplot(3,1,i); plot(time,yout(:,i+24),'r',t_out,ye(:,3*(i+6)-2),'--b');
        xlim([0,Tmax])
        ylim([-3,3])
        xlabel('Time (sec.)');
        ylabel('abs. acc. (m/s2)')
        title(['Hachinohe// floor' int2str(i+6) ' //Max abs. acc. ='...
              num2str(max(yout(:,i+24))) 'm']);
    end
    legend('Shear Model','Finite Element Model')

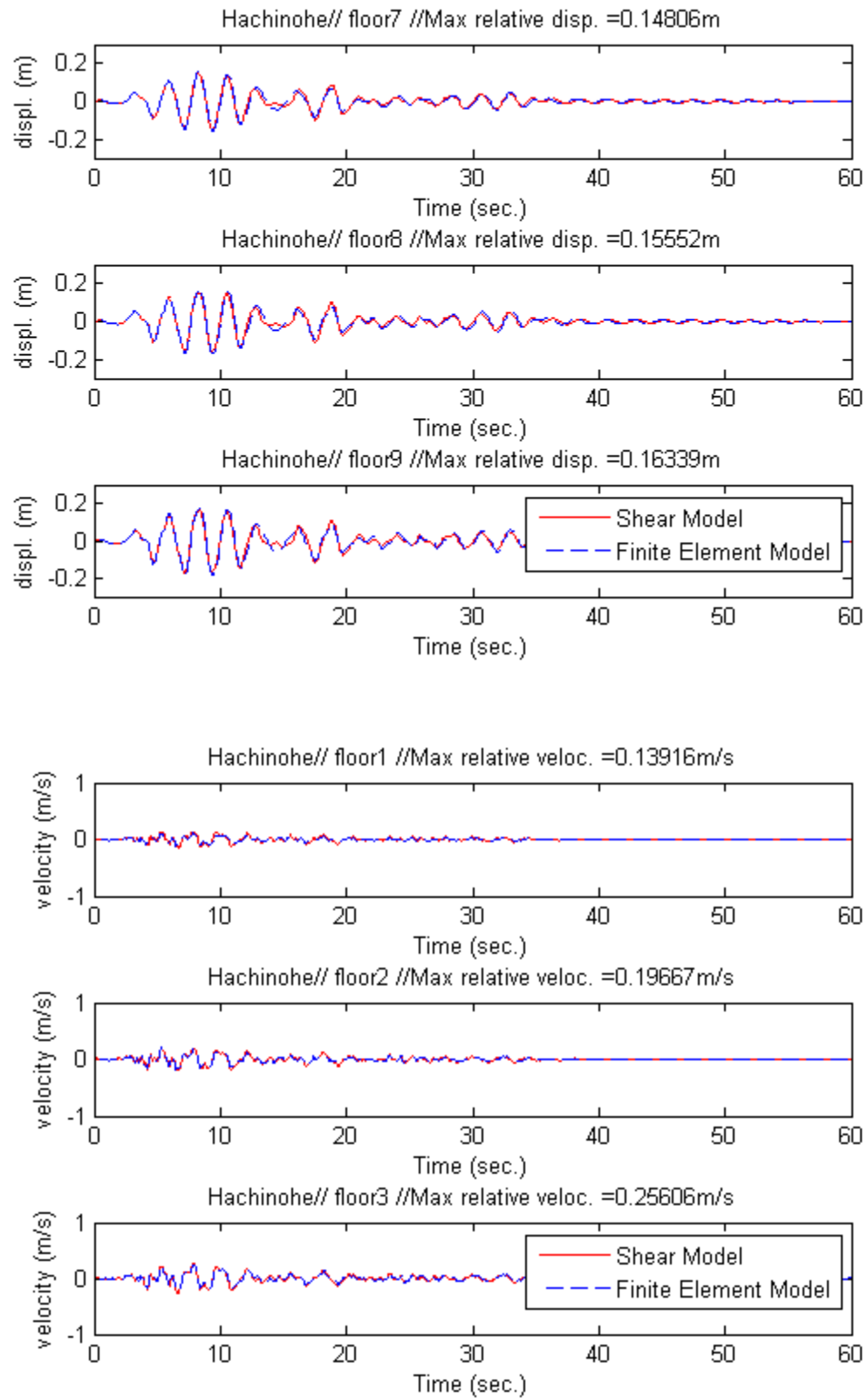
    for i = 1:9
        rmsHa(i,1) = (1/np)*sqrt(sum((yout(:,i)-ye(:,3*i)).^2)); %RMS Error Hachinohe
        rmsHa(i,2) = (1/np)*sqrt(sum((yout(:,i+9)-ye(:,3*i-1)).^2)); %RMS Error Hachinohe
        rmsHa(i,3) = (1/np)*sqrt(sum((yout(:,i+18)-ye(:,3*i-2)).^2)); %RMS Error Hachinohe
        PRh(i,2) = max(yout(:,i));
        PRh(i,1) = max(ye(:,3*i));
    end
    PREr = abs(PRh(:,1)-PRh(:,2))./(PRh(:,2))*100;
    figure(22)
    subplot(311)
    bar(rmsHa,'group');
    legend('Rel. Disp.','Rel. Vel.','Abs. Acc.')
    ylabel('RMS Error')
    title('RMS Error 50% intensity Hachinohe')
    subplot(312)
    bar(PRh,'group');
    ylabel('Peak Response (m)')
    title('Peak Response')
    legend('Finite Element Model','Shear Model')
    subplot(313)
    bar(PREr); xlabel('Floor No. ');
    ylabel('Peak Response Error(%)')
    title('Peak Response Error')

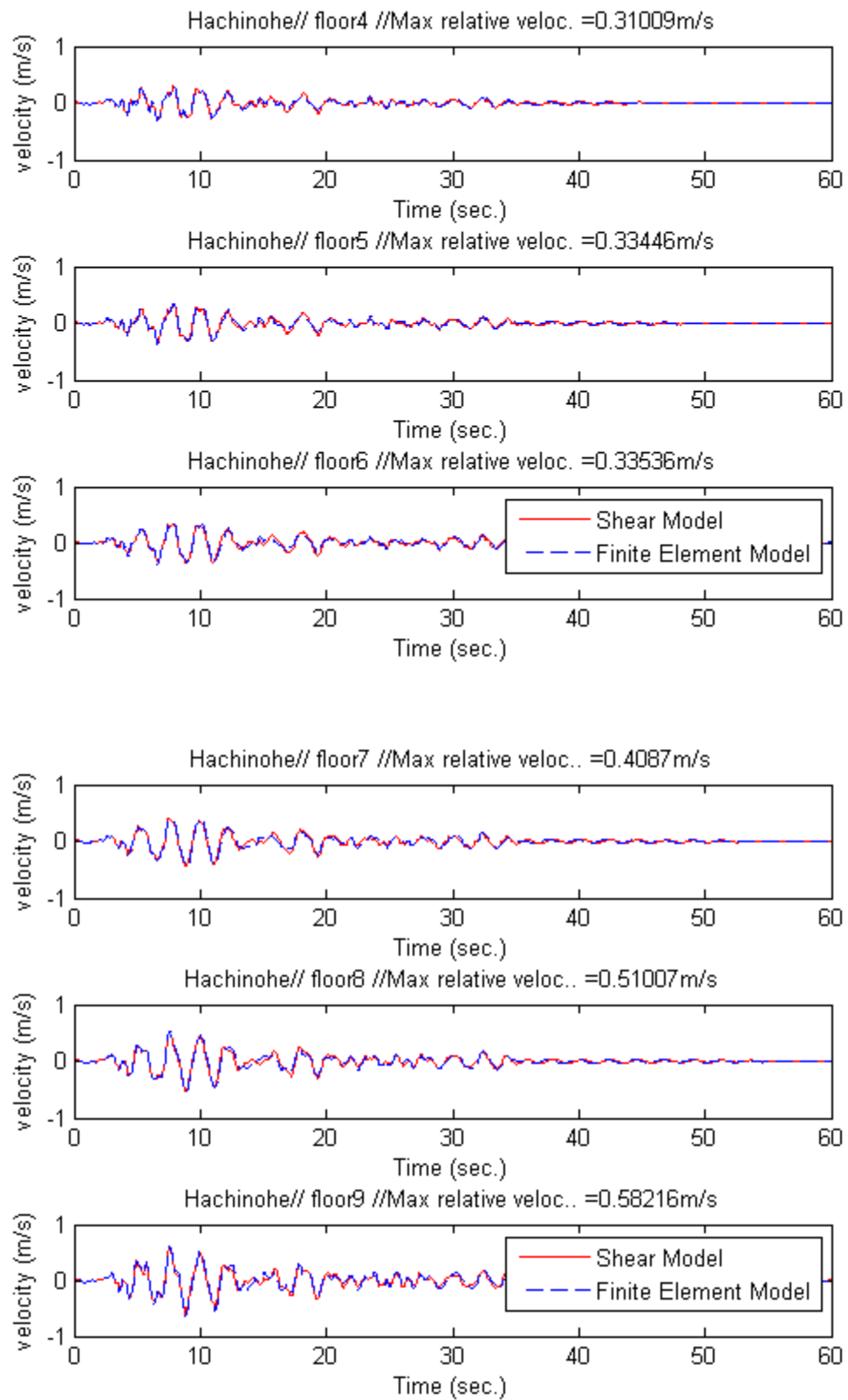
```

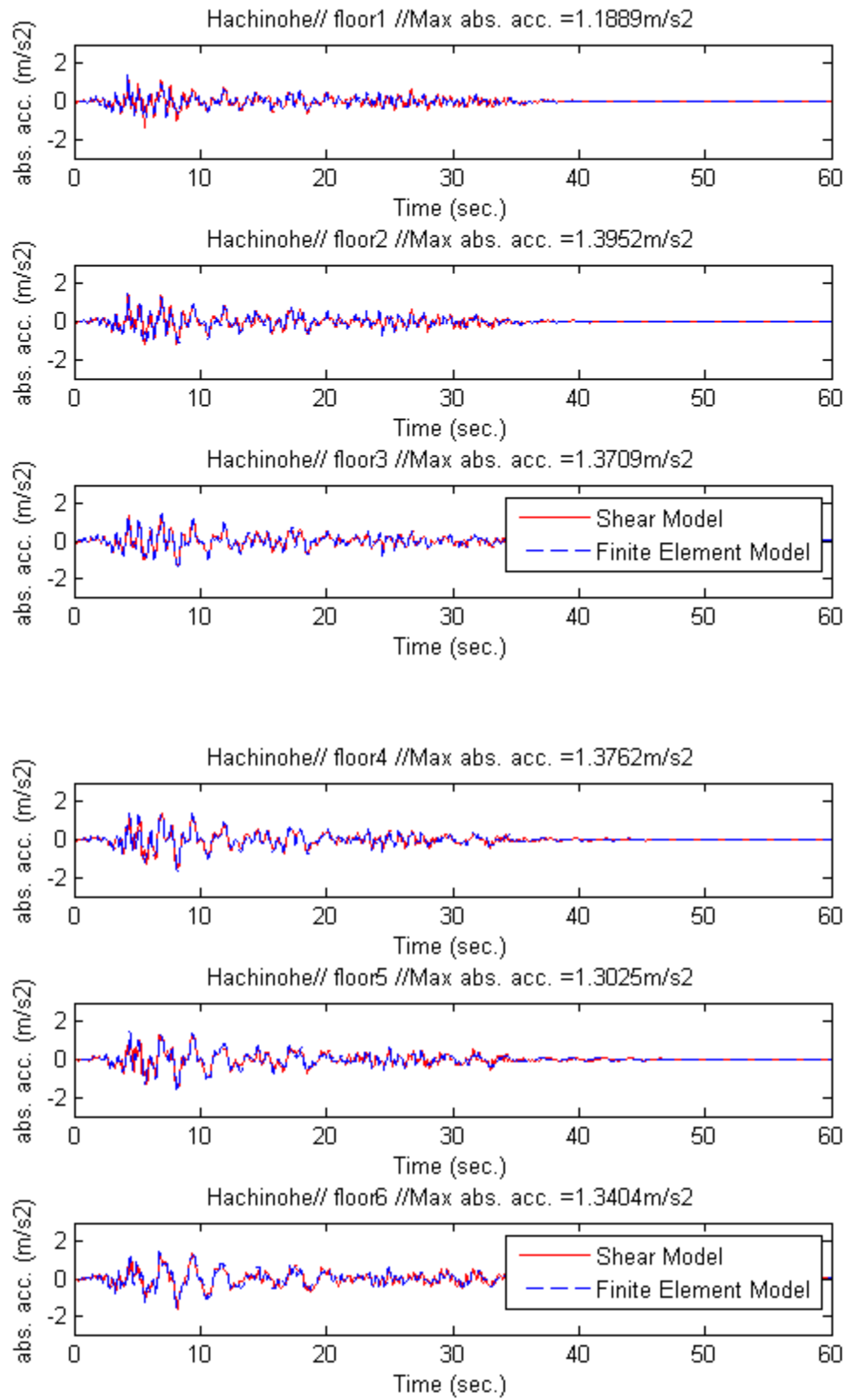
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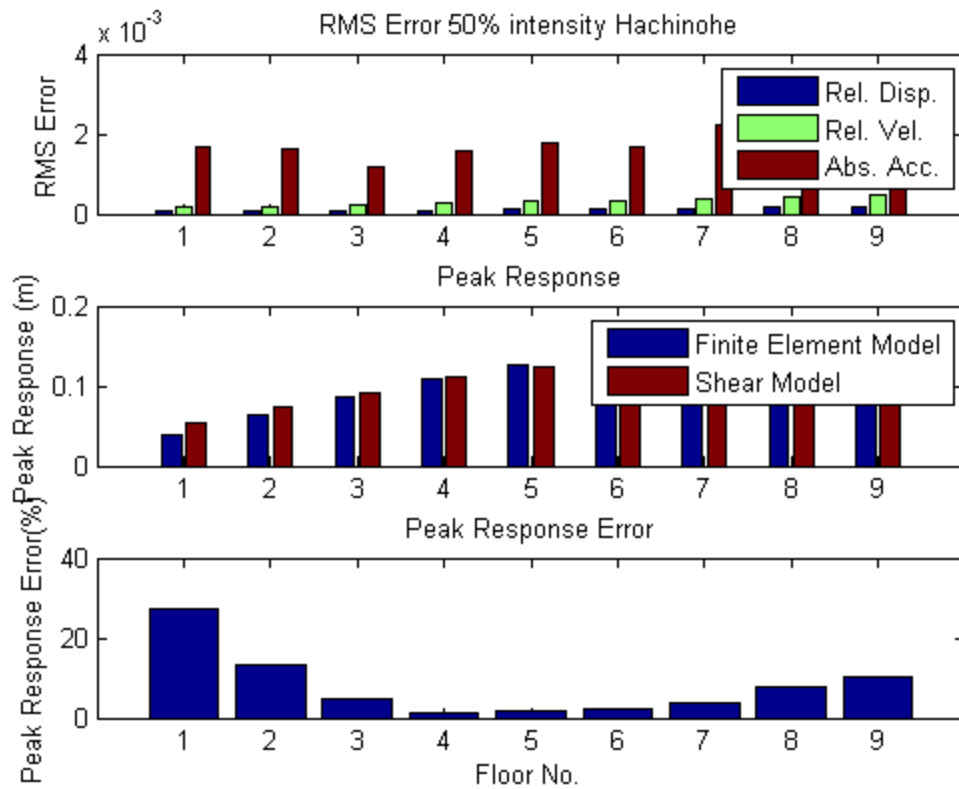
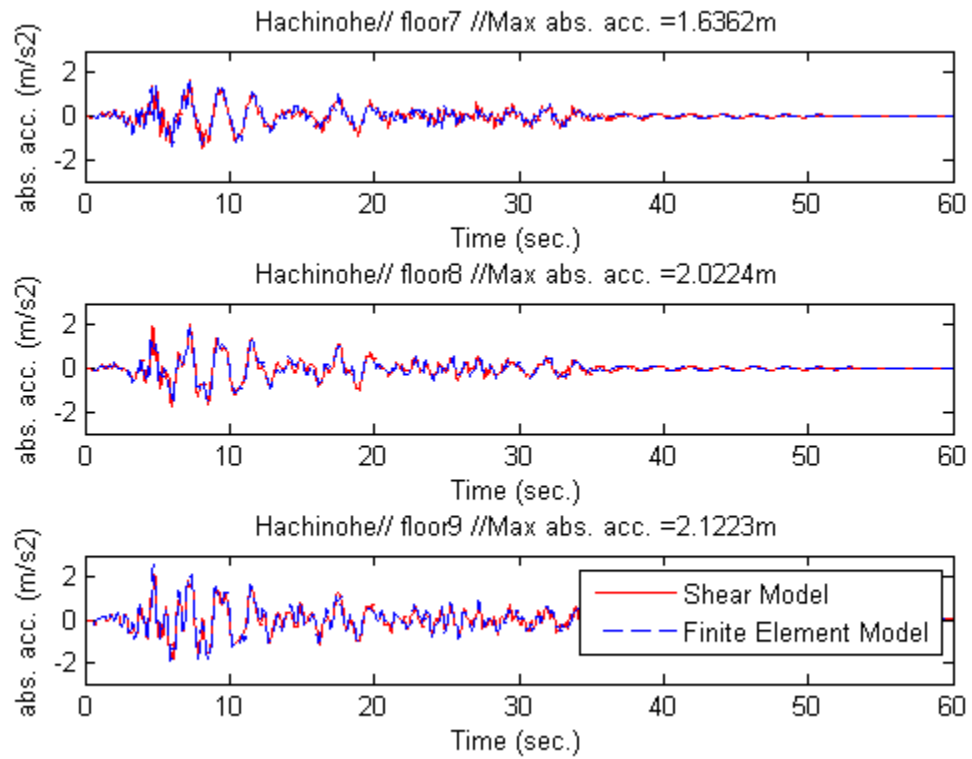












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# Mass Variation

```
load elcentro
t = e(:,1); int = 0.5; % Intensity
zg = int*e(:,2); %Ground Acceleration

load Low
AA = [zeros(N) eye(N);-inv(M)*K -inv(M)*C];
BB = [zeros(N,1);-ones(N,1)]; % earthquake input
CC = [eye(N) zeros(N); zeros(N) eye(N);-inv(M)*K -inv(M)*C];
DD = zeros(3*N,1);% earthquake input
sim('Model');
Y(:,1:27) = yout;

load Med
AA = [zeros(N) eye(N);-inv(M)*K -inv(M)*C];
BB = [zeros(N,1);-ones(N,1)]; % earthquake input
CC = [eye(N) zeros(N); zeros(N) eye(N);-inv(M)*K -inv(M)*C];
DD = zeros(3*N,1);% earthquake input
sim('Model');
Y(:,28:54) = yout;

load High
AA = [zeros(N) eye(N);-inv(M)*K -inv(M)*C];
BB = [zeros(N,1);-ones(N,1)]; % earthquake input
CC = [eye(N) zeros(N); zeros(N) eye(N);-inv(M)*K -inv(M)*C];
DD = zeros(3*N,1);% earthquake input
sim('Model');
Y(:,55:81) = yout;

figure(23)
for i = 1:3
    subplot(4,1,i); plot(time,Y(:,i), 'r',time,Y(:,i+27), 'b',time,Y(:,i+54), 'k');
    ylim([-0.4,0.4])
    xlabel('Time (sec.)');
    ylabel('displ. (m)')
    title(['Elcentro// floor' int2str(i)]);
    mdis(i,1:3) = max([Y(:,i),Y(:,i+27),Y(:,i+54)]); %Max Displacement
end
xlabel('Time (sec.)');
legend('-10%', 'M', '+10%')
subplot(4,1,4); bar(mdis(1:3,:), 'group')
xlabel('Floors (1-3)'); ylabel('Max. Displacement(m)')
legend('-10%', 'M', '+10%')

figure(24)
for i = 1:3
    subplot(4,1,i); plot(time,Y(:,i+3), 'r',time,Y(:,i+30), 'b',time,Y(:,i+57), 'k');
    ylim([-0.4,0.4])
    ylabel('displ. (m)')
    title(['Elcentro// floor' int2str(i+3)]);
    mdis(i+3,1:3) = max([Y(:,i+3),Y(:,i+30),Y(:,i+57)]); %Max Displacement
end
xlabel('Time (sec.)');
legend('-10%', 'M', '+10%')
subplot(4,1,4); bar(mdis(4:6,:), 'group')
xlabel('Floors (4-6)'); ylabel('Max. Displacement(m)')
legend('-10%', 'M', '+10%')

figure(25)
for i = 1:3
    subplot(4,1,i); plot(time,Y(:,i+6), 'r',time,Y(:,i+33), 'b',time,Y(:,i+60), 'k');
    ylim([-0.4,0.4])
```

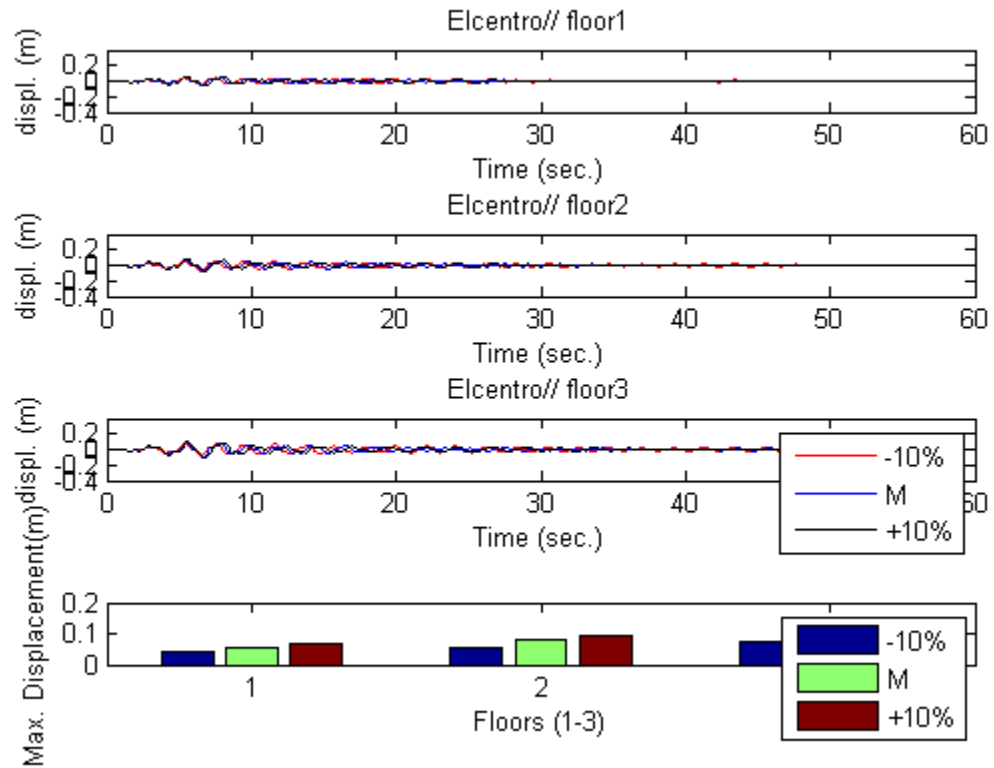
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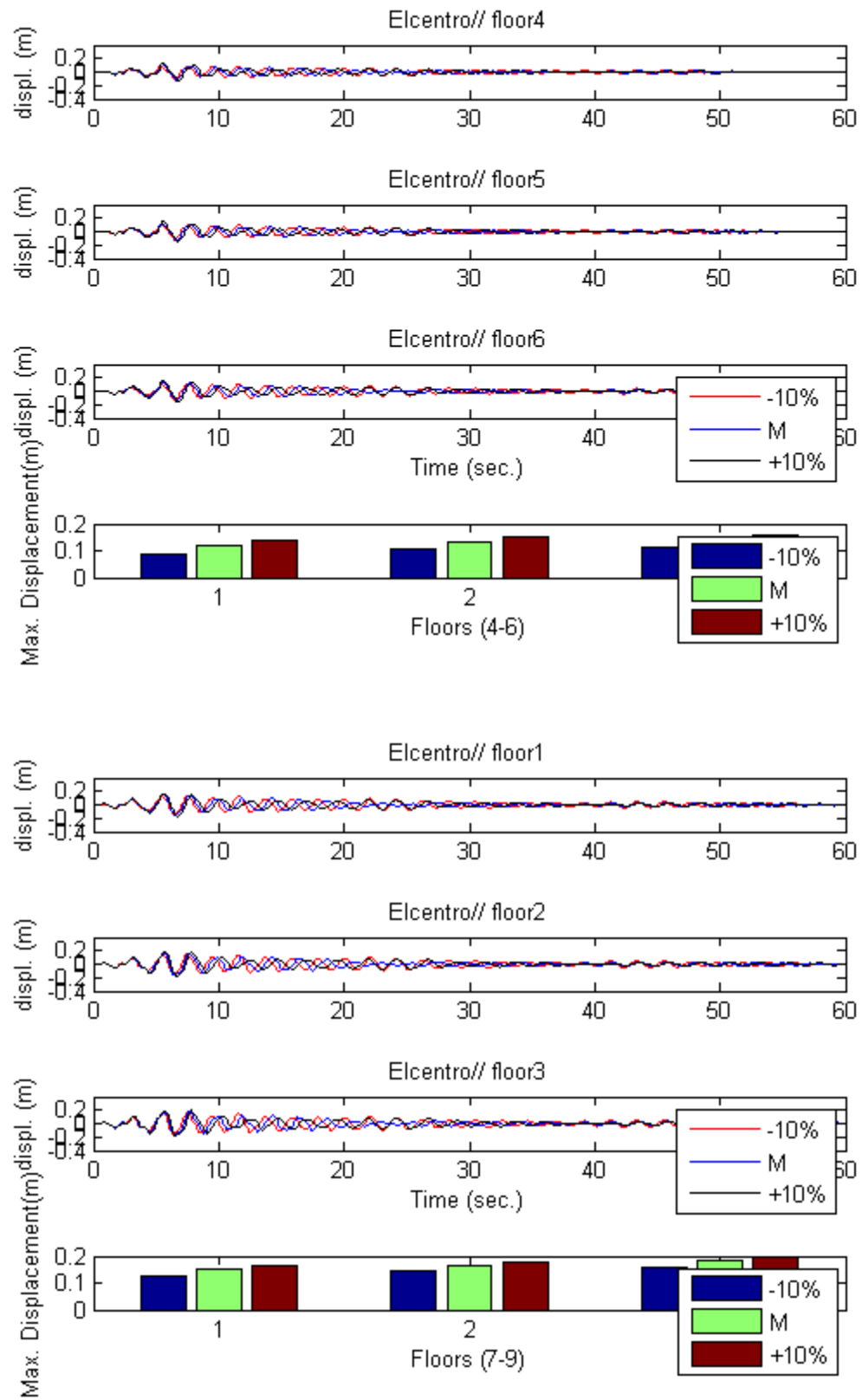
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```

        ylabel('displ. (m)')
        title(['Elcentro// floor' int2str(i)]);
        mdis(i+6,1:3) = max([Y(:,i+6),Y(:,i+33),Y(:,i+60)]); %Max Displacement
    end
    xlabel('Time (sec.)');
    legend('-10%', 'M', '+10%' )
    subplot(4,1,4); bar(mdis(7:9,:), 'group')
    xlabel('Floors (7-9)'); ylabel('Max. Displacement(m)')
    legend('-10%', 'M', '+10%' )

```





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