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A Tracking Error-Based Adaptive Compensation Scheme for Real-Time Hybrid Simulation

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ABSTRACT

Real-time hybrid simulation combines experimental testing and numerical simulation, and thus provides a viable alternative for the dynamic testing of civil engineering structures. Actuator delay compensation is vital to achieve a stable and reliable real-time hybrid simulation. An adaptive inverse compensation scheme for mitigating the effects of actuator delay is presented in this paper. A tracking-error based adaptive control law is developed to adapt a compensation parameter to minimize actuator control error due to an unknown time-varying actuator delay. Real-time hybrid simulations of a single-degree-of-freedom moment resisting frame with an elastomeric damper are conducted to experimentally demonstrate the effectiveness of the proposed adaptive compensation method. The adaptive compensation scheme is shown to improve actuator tracking capability by effectively negating a poorly estimated actuator delay.

INTRODUCTION

Hybrid simulation is an experimental method which investigates the dynamic response of a structure using a hybrid model comprised of both experimental (physical) and analytical (numerical) substructures. The method provides a viable alternative for large- or full-scale testing of civil engineering structures. Many load-rate dependent seismic devices have recently been developed to enhance the seismic performance of structural systems [Soong and Spencer 2002]. The development of performance-based design procedures for structures with these devices requires that the performance of the structural system with the devices be evaluated and the design procedure be verified by conducting tests at a real-time scale. Hybrid simulation has therefore been extended to real-time hybrid simulation to achieve these requirements in an economical and effective manner [Nakashima *et al.*, 1992]. Unlike conventional hybrid simulation [Dermitzakis and Mahin, 1985], the command displacements in a real-time hybrid simulation are imposed by the servo-hydraulic actuator(s) at a real-time scale. Due to inherent servo-hydraulic dynamics, a time delay in the restoring force of the experimental substructure can be introduced in a real-time hybrid simulation. The effect of actuator delay for real-time hybrid simulation has been investigated by researchers [Wallace *et al.*, 2005; Chen and Ricles, 2008a]. These studies indicate that actuator delay will destabilize the real-time simulation if not compensated properly.

Actuator delay compensation is often used in a real-time hybrid simulation to minimize the effect of actuator delay. Horiuchi *et al.* [1999, 2001] proposed compensation schemes based on a polynomial extrapolation and an assumption of linear structural acceleration, respectively. Other compensation methods also include the derivative feedforward method [Mercan, 2007]. Chen [2007] proposed an inverse compensation scheme for actuator delay compensation for real-time hybrid simulation based on a simplified model for servo-hydraulic actuator response. These methods are developed for a constant actuator delay. Applying these methods in real-time hybrid simulation requires an accurate estimate of actuator delay. Accurately estimating an actuator delay however can be difficult in actual practice. Moreover, the actuator delay might vary during the simulation due to the nonlinearities in the experimental substructure and the servo-hydraulic system.

Compensation methods for unknown or time-varying delay have also been developed. Darby *et al.* [1999] proposed an online procedure to estimate and compensate for actuator delay during a real-time hybrid simulation. Bonnet *et al.* [2007] applied the model reference adaptive minimal control synthesis (MCS) procedure to real-time hybrid simulation. In this paper a tracking-error based adaptive inverse compensation method is proposed to minimize the effect of an inaccurately estimated or a time varying actuator delay. The proposed adaptive compensation method is experimentally evaluated by large-scale real-time hybrid simulations of a single-degree-of-freedom (SDOF) moment resisting frame (MRF) with an elastomeric damper. Actuator control is shown to be significantly improved using the adaptive inverse compensation method, even with a poorly estimated actuator delay, to enable a reliable real-time hybrid simulation to be achieved.

ACTUATOR DELAY MODELING AND INVERSE COMPENSATION

For the real-time hybrid simulation of a SDOF structural model, as shown in Figure 1, the equation of motion can be written as

$$m \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + r^a(t) + r^e(t) = F(t) \tag{1}$$

where m and c are the mass and the inherent viscous damping of the SDOF structure, respectively; $\dot{x}(t)$ and $\ddot{x}(t)$ are the velocity and acceleration responses of the structure, respectively; $F(t)$ is the predefined external excitation force; and $r^a(t)$ and $r^e(t)$ are restoring forces of the analytical substructure (i.e., the moment resisting frame) and the experimental substructure (i.e., the elastomeric damper), respectively.

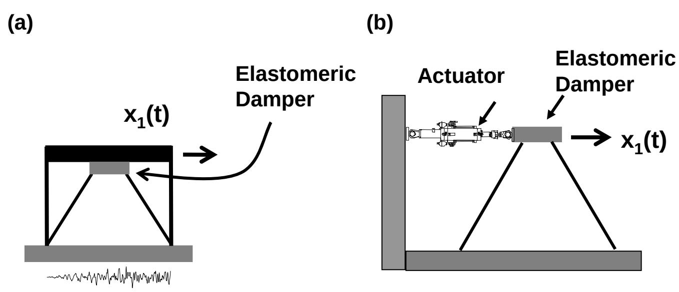

FIGURE 1 - (a) SDOF WITH ELASTOMERIC DAMPER, (b) EXPERIMENTAL SUBSTRUCTURE

A numerical integration algorithm is typically used in a real-time hybrid simulation to solve the temporally discretized form of (1) for structural response (displacement, velocity and acceleration). Using an explicit integration algorithm, the displacement x_{i+1} at time step $i+1$ is obtained based on feedback restoring forces r_i^a and r_i^e for the substructures at time step i. The displacement response x_{i+1} for the $(i+1)^{th}$ time step is then imposed to the substructures in order to obtain the restoring forces r_{i+1}^{a} and r_{i+1}^{e} to proceed to determine the displacement x_{i+2}. For the SDOF structure in Figure 1, the command displacements for the experimental and analytical substructures at time step $i+1$ are the same as x_{i+1}. To ensure a smooth actuator response, a ramp generator is used to interpolate the command displacement x_{i+1} over the integration time step Δt. The time step Δt is typically an integer multiple of the actuator's servo-controller sampling time δt. For a linear ramp generator, the command displacement sent to the servo-controller is interpolated as

$$d_{i+1}^{c(j)} = \frac{j}{n} \cdot (x_{i+1} - x_i) + x_i \quad (2)$$

In (2) j is the substep index for the interpolation substep of the ramp generator within the time step and ranges from 1 to n, where n is the integer ratio of $\Delta t/\delta t$. $d_{i+1}^{c(j)}$ in (2) is the displacement command for the servo-hydraulic actuator at the j^{th} substep of the $(i+1)^{th}$ time step.

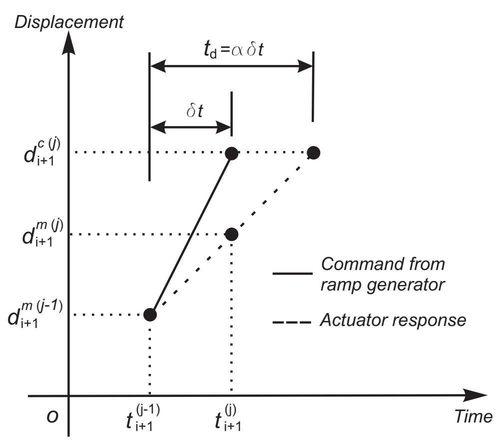

FIGURE 2 - ACTUATOR RESPONSE UNDER TIME DELAY

Due to servo-hydraulic dynamics that results in actuator delay, the servo-hydraulic actuator will reach a delayed measured response $d_{i+1}^{m(j)}$ instead of the command displacement $d_{i+1}^{c(j)}$. For a time interval of δt, which is typically 1/1024 *sec.* for state-of-art servo-controllers, the actuator response can be idealized as a linear response, as shown in Figure 2. The duration for the actuator to achieve the command displacement $d_{i}^{c(j)}$ is t_d and designated as $\alpha\,\delta t$. α is greater than 1.0 when a time delay exists in the actuator response. Assuming that the actuator achieves the measured displacement $d_{i+1}^{m(j-1)}$ at the end of the $(j\text{-}1)^{th}$ substep during the $(i+1)^{th}$ integration time step, and using the linear actuator response shown in Figure 2, the measured displacement response $d_{i+1}^{m(j)}$ at the end of the j^{th} substep of this time step can be expressed as

$$d_{i+1}^{m(j)} = d_{i+1}^{m(j-1)} + \frac{1}{\alpha} \cdot (d_{i+1}^{c(j)} - d_{i+1}^{m(j-1)}) \quad (3)$$

Applying the discrete z-transform [Ogata, 1995] to (3) leads to a discrete transfer function $G_d(z)$ that relates the measured actuator response $d_{i+1}^{m(j)}$ to the command displacement $d_{i+1}^{c(j)}$, where

$$G_d(z) = \frac{X^m(z)}{X^c(z)} = \frac{z}{\alpha \cdot z - (\alpha - 1)} \tag{4}$$

In (4), z is the complex variable in discrete z-domain, and $X^m(z)$ and $X^c(z)$ are the discrete z-transforms of $d_{i+1}^{m(j)}$ and $d_{i+1}^{c(j)}$, respectively. Chen and Ricles [2008a] proposed to use the inverse of the simplified actuator delay model in (4) for actuator delay compensation to improve actuator control in a real-time hybrid simulation. The method is referred to as the *inverse compensation method*, where the discrete transfer function $G_c(z)$ for the method can be written as

$$G_c(z) = \frac{X^p(z)}{X^c(z)} = \frac{\alpha \cdot z - (\alpha - 1)}{z} \tag{5}$$

where $X^p(z)$ is the discrete z-transform of the predicted displacement $d_{i+1}^{p(j)}$ to be sent to the servo-hydraulic actuator controller to compensate for actuator delay associated with the value of α for the j^{th} substep at time step i+1 in a real-time simulation. Chen *et al.* [2009] applied inverse compensation to the real-time hybrid simulation of structures with an elastomeric damper. Good actuator tracking was observed when an accurate value of α was used.

FORMULATION FOR ADAPTIVE INVERSE COMPENSATION

An accurate estimate of actuator delay, i.e., the value of α in (5), may not be available when applying the inverse compensation method for a real-time hybrid simulation, leading to an under- or over-compensation. To minimize the effect of an inaccurately estimated and time varying actuator delay, an adaptive inverse compensation method is developed in this paper, which can be formulated as

$$G_c(z) = \frac{X^p(z)}{X^c(z)} = \frac{(\alpha_{es} + \Delta\alpha) \cdot z - (\alpha_{es} + \Delta\alpha - 1)}{z} \tag{6}$$

In (6) α_{es} is the estimated actuator delay and $\Delta\alpha$ is an evolutionary variable for the adaptive inverse compensation method with an initial value of zero. An adaptive control law is used to determine $\Delta\alpha$ based on a tracking indicator (*TI*) described below, where $\Delta\alpha$ is defined as

$$\Delta\alpha(t) = k_p \cdot TI(t) + k_i \cdot \int_0^t TI(\tau)dt \tag{7}$$

In (7) k_p and k_i are proportional and integrative adaptive gains, respectively. The calculation of *TI* at each time step is formulated as [Mercan 2007]

$$TI_{i+1}^{(j)} = 0.5\left(A_{i+1}^{(j)} - TA_{i+1}^{(j)}\right) \tag{8}$$

$A_{i+1}^{(j)}$ and $TA_{i+1}^{(j)}$ in (8) are associated with the j^{th} substep of time step i+1 and calculated as

$$A_{i+1}^{(j)} = A_{i+1}^{(j-1)} + 0.5\left(d_{i+1}^{c(j)} + d_{i+1}^{c(j-1)}\right)\left(d_{i+1}^{m(j)} - d_{i+1}^{m(j-1)}\right) \tag{9a}$$

$$TA_{i+1}^{(j)} = TA_{i+1}^{(j-1)} + 0.5\left(d_{i+1}^{m(j)} + d_{i+1}^{m(j-1)}\right)\left(d_{i+1}^{c(j)} - d_{i+1}^{c(j-1)}\right) \tag{9b}$$

At the beginning of the test, *A* and *TA* have initial values of zero. Mercan [2007] showed that a positive rate of change of the *TI* corresponds to an actuator response lagging behind the command displacement, while a negative rate of change indicates a leading actuator response. A zero rate of change of the *TI* implies no actuator control error, i.e., the actuator measured and command displacements are equal to each other.

Equation (7) gives the adaptation of *Δα* in continuous form. For the purpose of implementation, (7) needs to be expressed in discrete form, which is

$$\Delta\alpha(z) = k_p \cdot TI(z) + k_i \cdot \frac{\delta t}{z-1} \cdot TI(z) \tag{10}$$

where $\Delta\alpha(z)$ and $TI(z)$ are the discrete z-transforms of *Δα* and the *TI*, respectively. Generally, a larger value of k_p results in a faster response and a larger oscillation of the evolutionary variable *Δα*, while increasing the integration gain k_i reduces the oscillation. From numerical simulations of real-time hybrid simulations it is recommended that the integrative gain k_i be selected to be one tenth of the proportional gain k_p. Figure 3 shows the schematic representation of the adaptive inverse compensation method for real-time hybrid simulation that was implemented into the real-time integrated control system at the NEES Real-Time Multi-Directional (RTMD) Facility at Lehigh University.

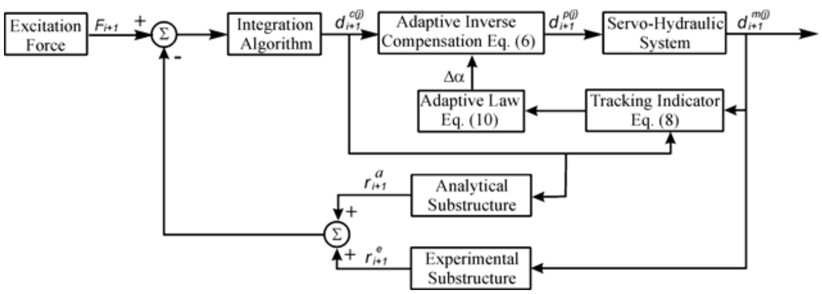

FIGURE 3 – SCHEMATIC IMPLEMENTATION FOR ADAPTIVE INVERSE COMPENSATION METHOD

REAL-TIME HYBRID SIMULATIONS WITH ADAPTIVE INVERSE COMPENSATION

To experimentally evaluate the performance of the proposed adaptive inverse compensation method, laboratory tests involving large scale real-time hybrid simulations are conducted using the SDOF MRF with an elastomeric damper. The SDOF MRF (without the elastomeric damper) has a mass of 503.4 metric tons, an elastic natural frequency of 0.77 Hz, and an inherent viscous damping ratio ζ of 0.02. Figure 4 shows the experimental setup, which consists of the experimental substructure (elastomeric damper), servo-hydraulic actuator with a support and roller bearings, and two reaction frames. The elastomeric damper is manufactured from a Butyl blend of rubber and has the characteristics of an elastomeric material at small deformation amplitudes, with friction dominating the behavior at larger amplitudes [Kontopanos, 2006]. The actuator in the experimental setup imposes the inter-story displacement to the damper, where the axial stiffness of the loading stub represents the horizontal stiffness of the diagonal braces in the structure. The servo-hydraulic actuator controller for the experimental setup consists of a digital PID controller with the proportional gain of 20, integral time constant of 5.0 *sec.* resulting in an integral gain of 4.0, differential gain of 0 and a roll-off frequency of 39.8 Hz. The large servo-hydraulic actuator has a 1700 kN maximum force capacity with a 500 mm stroke. Two servo-valves, each with a flow capacity of 2500 liters/min, are mounted on the actuator.

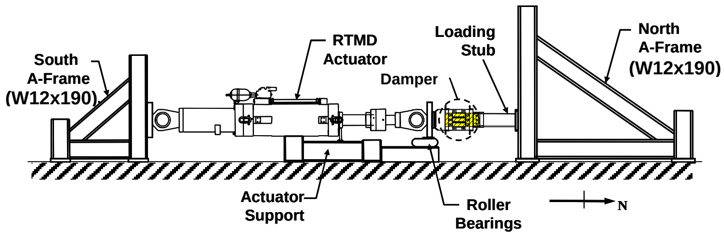

FIGURE 4 - EXPERIMENTAL SETUP FOR REAL-TIME HYBRID SIMULATION

The explicit CR integration algorithm [Chen and Ricles, 2008b] is used in the simulation, where the variations of displacement and velocity over the time step are defined as

$$\dot{x}_{i+1} = \dot{x}_i + \Delta t \cdot \alpha_1 \cdot \ddot{x}_i \tag{11a}$$

$$x_{i+1} = x_i + \Delta t \cdot \dot{x}_i + \Delta t^2 \cdot \alpha_2 \cdot \ddot{x}_i \tag{11b}$$

To incorporate the rate-dependent properties of the elastomeric damper in the real-time hybrid simulation, the integration parameters α_1 and α_2 are determined as

$$\alpha_1 = \alpha_2 = \frac{4m}{4 \cdot m + 2 \cdot \Delta t \cdot (c + c_{eq}) + \Delta t^2 \cdot (k_{eq} + k_a)} \tag{12}$$

where c_{eq} and k_{eq} are the equivalent damping and equivalent stiffness of the elastomeric damper, respectively. The values of k_{eq} and c_{eq} were determined from identification tests performed on the damper and are equal to 7.6 kN/mm and 0.64 kN-sec/mm, respectively. Chen *et al.* [2009] compared real-time hybrid simulations using the explicit CR algorithm with experiments using the HHT α-method with a fixed number of substep iterations [Shing, 2002]. Good comparisons were observed between the results, indicating that the CR algorithm can help achieve exceptional and reliable real-time hybrid simulation results.

The SDOF MRF analytical substructure is modeled using the Bouc-Wen model [Wen, 1980], whereby the restoring force of the MRF is defined as

$$r^a(t) = \eta \cdot k_a \cdot x^a(t) + (1-\eta) \cdot k_a \cdot x_y^a \cdot z(t) \tag{13}$$

In (13) x_y^a is the yield displacement of the analytical substructure; k_a is the linear elastic stiffness of the analytical substructure; η is the ratio of the post- to pre-yield stiffness of the analytical substructure; $x^a(t)$ is the displacement imposed on the analytical substructure by the integration algorithm; and $z(t)$ is the evolutionary parameter of the Bouc-Wen model governed by the following differential equation:

$$x_y^a \cdot \dot{z}(t) + \gamma \left|\dot{x}^a(t)\right| \cdot z(t) \cdot \left|z(t)\right|^{q-1} + \beta \cdot \dot{x}^a(t) \cdot \left|z(t)\right|^q - \dot{x}^a(t) = 0 \tag{14}$$

The dimensionless parameters γ, β and q in (14) control the shape of the hysteretic loop of the analytical substructure. The properties of the Bouc-Wen model for the analytical substructure are given below in Table 1.

k_a (KN/mm)	η	x_y^a (mm)	β	γ	q
11.76	0.01	10	0.55	0.45	2

TABLE 1- PARAMETERS OF THE BOUC-WEN MODEL FOR ANALYTICAL SUBSTRUCTURE

The time step Δt for real-time hybrid simulation is selected as 10/1024 sec. The N196E component of the 1994 Northridge earthquake recorded at Canoga Park was selected as the ground motion. To ensure a maximum displacement response of less than 30 mm in order not to damage the damper, the ground motion is scaled to have a maximum magnitude of acceleration of 0.322 m/s^2. To systematically evaluate the performance of the proposed adaptive compensation method, different values of α_{es}, k_p and k_i are used in the real-time hybrid simulations. Three different values for the estimated actuator delay constant α_{es} (15, 29 and 45) and three sets of the proportional adaptive gains (k_p=0, 0.2 and 0.4) are used. The case of k_p=0 represents a real-time hybrid simulation using conventional inverse compensation, i.e., $G_c(z)$ from (5). The value of k_i was always set equal to 0.1k_p. From tests, the actuator delay is known to be near a value of α_{es}=29. Therefore, values of α_{es}=15 and α_{es}=45 represent about a ±50% error in the estimate for actuator delay when compared with the actual actuator delay.

The real-time hybrid simulation results using the proposed adaptive compensation method with α_{es} equal to 15 are presented in Figure 5. The comparison of the command displacements and the measured actuator response is presented in Figure 5(a) for the simulation with k_p equal to 0.4. Good tracking can be observed in Figure 5(a). The difference between the command displacements and the actuator measured displacements (referred to as control error) is shown in Figure 5(b). The results for k_p=0.2 and k_p=0.4 have a maximum control error of 0.47 mm and 0.43 mm, respectively, while the maximum control error for k_p=0.0 is around 1.5 mm. It can thus be observed from the results for k_p=0.0 that α_{es}=15 is a poor estimate of actuator delay for the hybrid simulation. The simulation with k_p=0.4 has the smallest control error for all the simulations, where the maximum magnitude of control error of 0.43 mm corresponds to 1.21% of the maximum magnitude of the command displacement. The tracking indicator for the simulation is presented in Figures 5(c). It can be observed that simulations with adaptive compensation have significantly smaller values for the *TI* compared with simulations that don't utilize adaptive compensation (i.e., simulations with k_p = 0). The *TI* in Figure 5(c) increases initially due to the under-estimation of actuator delay (i.e., α_{es}=15) and then decreases when the adaptive compensation takes effect. The adaptation of the evolutionary variable $\Delta\alpha$ for the real-time hybrid simulations is presented in Figure 5(d). The adaptive mechanism is shown to make notable adjustments in the inverse compensation parameter.

The displacement history of the real-time hybrid simulation with k_p=0.4 and α_{es}=15 has maximum and minimum values of about 34.5 mm and -10.3 mm, respectively. Yielding of the analytical substructure occurred, beginning at around 15 sec., leading to a residual drift of 4 mm at the end of the test. The force-deformation responses of the analytical and experimental substructures are presented in Figure 6. The restoring forces developed in the two substructures are shown to be approximately the same value, whereby the elastomeric damper resisted from 44% to 55% of the story shear of the structure. Energy dissipation is shown to have occurred in both the elastomeric damper and the MRF.

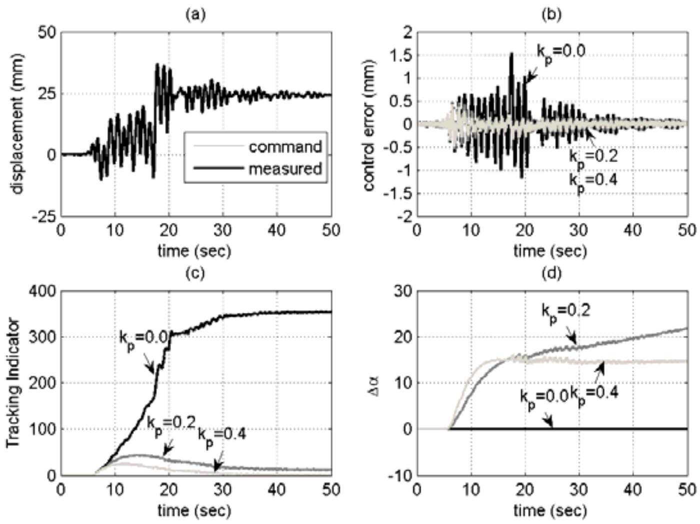

FIGURE 5 - REAL-TIME HYBRID SIMULATION RESULTS WITH ADAPTIVE INVERSE COMPENSATION (α_{es}=15)

The real-time hybrid simulation results using adaptive inverse compensation with α_{es} equal to 29 are presented in Figure 7. The comparison of the command displacement and the measured actuator displacement for the real-time hybrid simulation using adaptive compensation method with k_p=0.4 is presented in Figure 7(a). Good tracking can again be observed. The actuator control error is presented in Figure 7(b), where the results for the simulations using different values of k_p are shown. The results for k_p=0.2 and k_p=0.4 are nearly identical. A maximum magnitude of 0.18 mm, 0.16 mm, and 0.16 mm occurred in the simulations with k_p = 0, 0.2, and 0.4, respectively. The maximum control error of 0.18 mm and 0.16 mm correspond to about a 0.52% and 0.47% error in the command displacement. Therefore, α_{es}=29 is a good estimate of the actuator delay in the real-time hybrid simulation. Figure 7(c) shows the time history for the *TI* of the real-time hybrid simulations. The simulation with k_p=0 is observed to have a minimum value of -10, while the two with adaptive compensation (i.e., k_p=0.2 and 0.4) are observed to have the values of the *TI* equal to almost zero, indicating better actuator tracking for the real-time hybrid simulation is achieved when the proposed adaptive inverse compensation method is used. Figure 7(d) presents the time history of the evolutionary variable $\Delta\alpha$, which can be observed to have a small negative value for k_p=0.2 and 0.4. This implies that the adaptive mechanism adjusted for a slight over-compensation that exists when α_{es} equals 29.

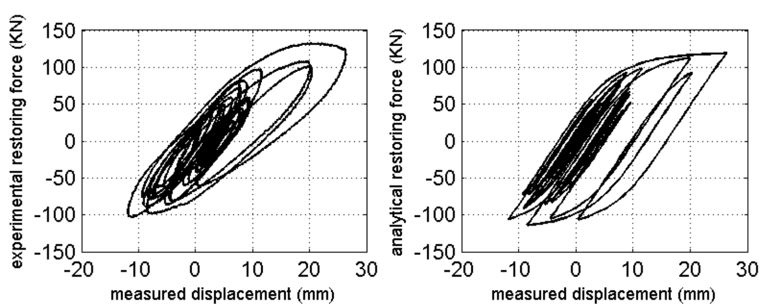

FIGURE 6 - HYSTERESIS OF ANALYTICAL AND EXPERIMENTAL SUBSTRUCTURES FOR REAL-TIME HYBRID SIMULATION (α_{es}=29, KP=0.4, KI=.04)

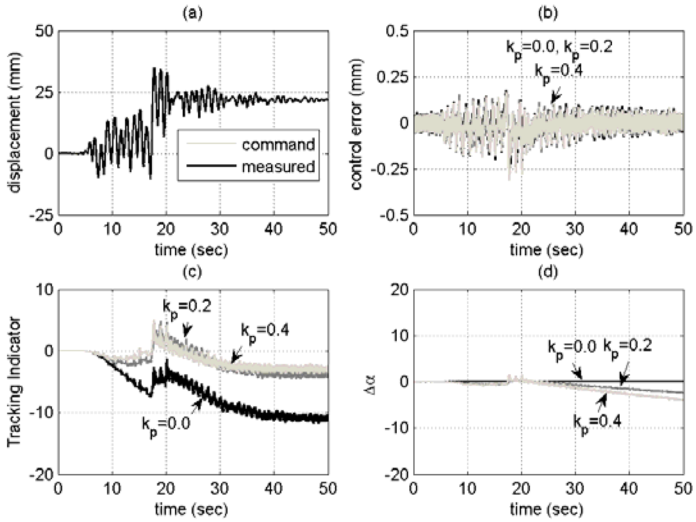

FIGURE 7 - REAL-TIME HYBRID SIMULATION RESULTS WITH ADAPTIVE INVERSE COMPENSATION (α_{es}=29)

The real-time hybrid simulation results using the proposed adaptive compensation method with α_{es} equal to 45 are presented in Figure 8. The comparisons of the command displacements and the measured actuator response in Figure 8(a) indicate good tracking for the simulation with k_p=0.4. The control error between the command displacements and the measured actuator responses are shown in Figure 8(b). The results for k_p=0.2 and k_p=0.4 have a maximum control error of 0.58 mm and 0.52 mm, respectively, while for k_p=0.0 the maximum control error is 1.12 mm. Therefore the value of 45 is a poor estimate of actuator delay for the inverse compensation. The simulation with k_p=0.4 again has the smallest control error for all the simulations, where the maximum magnitude of control error of 0.52 mm corresponds to 1.50% of the maximum magnitude of the command displacement. The values of the tracking indicators for the simulations are presented in Figure 8(c), where simulations with adaptive compensation again have significantly smaller values for the *TI* compared to simulations that don't utilize adaptive compensation (i.e., simulations with $k_p = 0$). An initial decrease can be first observed for the *TI* due to the over-estimation of actuator delay before the *TI* approaches zero because of the adaptive compensation. The adaptation of $\Delta\alpha$ in Figure 8(d) indicates that the adaptive mechanism makes notable adjustments in the inverse compensation parameter.

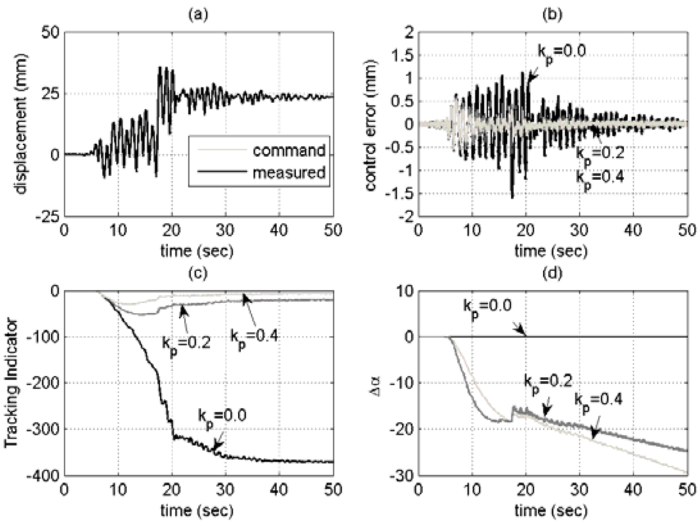

FIGURE 8 - REAL-TIME HYBRID SIMULATION RESULTS WITH ADAPTIVE INVERSE COMPENSATION (α_{es}=45):

SUMMARY AND CONCLUSIONS

Inherent servo-hydraulic dynamics in an actuator leads to actuator delay which has to be properly compensated to achieve reliable experimental results for a real-time hybrid simulation. An adaptive compensation method is proposed in this paper to achieve accurate actuator control using inverse compensation. An adaptive control law is developed for the compensation parameter using a tracking indicator that is based on the actuator tracking error. The compensation parameter is adapted to minimize the effect of inaccurately estimated and possibly time-varying actuator delay during a real-time hybrid simulation.

Real-time hybrid simulations of a SDOF MRF with an elastomeric damper are conducted to experimentally demonstrate the effectiveness of the proposed adaptive inverse compensation method. Different values of estimated actuator delay are used. The tracking capability of the servo-hydraulic actuator is shown to be greatly improved and the actuator control error is significantly reduced when the adaptive inverse compensation method is used, compared with the simulation results obtained using the conventional inverse compensation method. The proposed adaptive scheme is shown to require minimal information of the actuator delay before a real-time hybrid simulation is performed, while enabling exceptional experimental results to be achieved.

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